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FOR

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THE
STEAM ENGINE
FOR
PRACTICAL MEN.

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STEAM ENGINE

FOR

PRACTICAL MEN.

CONTAINING

A THEORETICAL INVESTIGATION OF THE VARIOUS RULES

GIVEN IN THE WORK,

AND SEVERAL USEFUL TABLES.

JOINTLY WRITTEN

BY

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PREFACE.

THE books which till now have appeared written for practical men, have either been exclusively devoted to practice, or have contained both theory and practice intermixed. Both systems we consider vicious, the first as not affording to the practical man the means of investigating by himself the formulæ from which the rules have been deduced, the second as puzzling the purely practical mind by the introduction of intricate formulæ where he seeks only for practice and practical rules. In the present treatise, we are inclined to think all this has been obviated.

The First Part does not contain a single formula; the practical man will here find nothing but rules and their immediate application, an example, or more, being always given after each rule.

The Second Part has been exclusively devoted to the theoretical investigation of the rules given in the First Part.

And in the Appendix we have mixed both theory and practice, on subjects of minor importance.

The numerous accidents that have lately happened by the frequent explosions of steam boilers, has led us to enter rather fully into the discussion of this important subject. We wish to call the attention of the public to the fact, that boilers rarely, if ever, explode except through the want of water in them. The well-known engineer, *Mr. Fairbairn,* of Manchester, has lately written an

excellent Pamphlet, coinciding almost wholly with our views on this subject.

On the Steam Jet, we think we have said enough to show its importance—besides, this subject has been alrèady sufficiently discussed before Committees of both Houses of Parliament. Nevertheless, we cannot refrain from expressing our surprise that the Admiralty, when the country is on the eve of a war, should not have paid due attention to the plan communicated to them by *Dr. Vacy*, in February, 1846, for employing horizontal funnels in war steamers, instead of the vertical ones in present use. He proposed to produce a draught through the furnaces by means of the Steam Jet. This plan, if carried into operation, would undoubtedly prevent any diminution of power in the engines in case the funnel should be carried away by shot, or by any other accident.

The eminent scientific men to whom we are indebted are *Woolhouse*, *Rawson*, *Sewell*, *Amos*, *Tate*, and *Holland*, one of the Admiralty engineers. The principal publications we have consulted are the *Mechanics' Magazine*, and the *Civil Engineer and Architect's Journal*, works containing the latest information concerning the discoveries and inventions that have taken place in every domain of Art or Science. We may also mention the *Philosophical Transactions* of the Royal Society of London, and some of the works of that able writer *Tredgold*.

Many excellent articles on subjects connected with the principles of heat and steam have appeared in the *Mechanics' Magazine*, under the signature A. H. Who this A. H. can be we are unable to say, but we must state, that as he has not hidden his talents *under a bushel*, we see no good reason why he should hide his name under it.

In the *Civil Engineer and Architect's Journal*, we have a similar case; many articles of surpassing merit have emanated from a gentleman that signs himself H. C. This is equally unintelligible to the masses, to us it is

not quite so mysterious; we are inclined to believe that H. C. is no other person than *Homersham Cox,* a gentleman well known as one of the ablest contributors to Practical Science in this country.

In conclusion, we may state, that at the end of the work we give Tables of various kinds, which will be found useful to our readers.

THE AUTHORS.

December 16*th,* 1853.

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STEAM ENGINE.

THE law discovered by Mariotte, and confirmed by Arago and Dulong, as far as up to pressures of 27 atmospheres, is the simplest that can be applied to the Steam Engine. It may be enunciated thus:—

If a given weight of steam or gas be made to vary its volume without changing its temperature, the elastic force of the steam or gas will vary in the inverse ratio of the volume it is made to occupy.

Thus, let ABCD be a cylinder, EF and HG any two positions of the piston, then the pressure in the position EF is to the pressure in the position HG as the space HDCG is to the space EDCF. Although this is by far the most simple law that we can apply, yet it is not strictly correct, but sufficiently so for the purpose of showing the great advantages that are to be gained by working steam expansively; the results, however, will be always greater than the truth, since, during the time the expansion is going on in the cylinder, the temperature does not remain exactly the same. We shall hereafter give in detail the various properties of steam given by Arago and Dulong, Gay-Lussac, Tredgold, Biot, Pambour, Pole, and others.

Work, or dynamical effect, as it is sometimes called,

supposes a body moved, and a resistance overcome: either of these, without the other, is insufficient to constitute work. The work, produced by a pressure moving a body through a certain space, is defined to be the product arising from multiplying the pressure by the space through which this pressure acts.

Unit of Work.

The unit of work, in this country, in terms of which we measure any amount of work, is the work done where a pressure of one pound is exerted through one foot, the pressure acting in the direction in which the space is described: if, instead of one pound being moved through one foot, it be moved through two feet, it is clear that the work is doubled, or that two units of work have been done.

The Unit of Work in reference to the Unit of Time.

The unit of work has been assumed to be 1 lb. raised 1 foot high. Let this be referred to 1 minute as the unit of time. Considered in this point of view, the unit of work will be represented by 1 lb. raised 1 foot high in one minute. Now, it is assumed that a horse is capable of doing 33,000 such units of work; *i. e.* he is capable of raising 33,000 lbs. 1 foot high in one minute, or 1 lb. 33,000 feet high; and this is called a horse's power, and is the unit of work in reference to the unit of time commonly used in this country. To determine, then, the number of horses' power consumed in any given work, we have only to divide the amount of that work by 33,000 times the number of minutes in which it is done.

Given the length of the stroke, the distance travelled by the piston before the steam is cut off, and the pressure at which the steam is admitted in the cylinder, to find the work done upon each square inch of the piston in one stroke.

Rule 1.—Multiply the pressure at which the steam is admitted, by the distance travelled by the piston before the steam is cut off; this gives the work done before expansion begins.

Divide the whole length of the stroke by the above-mentioned distance, and find the hyperbolic logarithm of the quotient. Multiply this hyperbolic logarithm by the work done before expansion, and the result is the work done after expansion begins.

Adding together the work done before to that done after expansion, we obtain the whole work done upon one square inch of the piston in one stroke.

To find the Horse Power.

Multiply the work done on one square inch in one stroke, by the area of the piston in square inches, and by the number of strokes per minute; this product divided by 33,000 gives the horse power.

To find the Load.*

Rule 2.—Having by the preceding rule found the whole work done on one square inch in one stroke, divide it by the length of the stroke, and the quotient will be the load upon one square inch of the piston in one stroke.

To find the Pressure at which the Steam is admitted.

Rule 3.— Divide the whole length of the stroke by the number of feet described by the piston before the steam is cut off, find the hyperbolic logarithm of this quotient, add unity to this logarithm, and multiply this sum by the number of feet described by the piston before the steam is cut off for a divisor, and for a dividend multiply the load by the length of the stroke. The quotient is the pressure in pounds per square inch at which the steam is admitted.

* The *load* is generally defined to be the mean pressure of the steam.

To find where the Velocity of the Piston is greatest.

Rule 4.—Divide the length of the stroke by the distance travelled by the piston before the steam is cut off; take the hyperbolic logarithm of the quotient, add unity to this logarithm for a divisor, and for a dividend take the length of the stroke.

The quotient will be the distance moved over by the piston when the velocity is greatest.

Example 1.—The pressure of steam upon the piston is 60 lbs. per square inch, the length of the stroke is 10 feet, the steam is cut off at $\frac{1}{5}$ of the stroke: find the number of units of work done upon each square inch of the piston.

$60 \times 2 = 120$ units of work done before expansion begins.

$$\text{Hyp. log. of } \frac{10}{2} = \text{hyp. log. of } 5 = 1{\cdot}6094379.$$

$1{\cdot}6094379 \times 120 = 193{\cdot}132548$, units of work done after expansion begins.

120·00 units of work before expansion.
193·13 units of work after expansion.

313·13 whole work done on one square inch in one stroke.

In the above example, to find the load.

$\frac{313{\cdot}13}{10} = 31{\cdot}313$, load in lbs. upon one square inch of the piston.

If the cylinder be 40 inches in diameter, then the area will be

$$40 \times 40 \times {\cdot}7854 = 1256{\cdot}64 \text{ square inches.}$$

The whole work done on one inch in one stroke with and without expansion being

313·13 lbs.,

the whole work on the piston of the above cylinder will be

$$1256{\cdot}64 \times 313{\cdot}13 = 393491{\cdot}68 \text{ lbs.}$$

The load upon each square inch of the piston being

31·313 lbs.,

the whole load upon the whole piston will therefore be

$$1256{\cdot}64 \times 31{\cdot}313 = 39349{\cdot}17 \text{ lbs.}$$

If the engine makes 20 strokes per minute, to find the horse power.

$$1256 \cdot 64 \times 313 \cdot 13 = 393491 \cdot 68 \text{ lbs.}$$

$$\frac{393491 \cdot 68 \times 20}{33000} = 238 \cdot 48 \text{ horse power.}$$

Example 2.—At what pressure per square inch must the steam be admitted when the load is 22 lbs. per square inch, the length of the stroke 5 feet, and the steam is cut off at 2 feet?

5 feet, length of the stroke.

2, number of feet described by the piston before the steam is cut off.

$$\text{Hyp. log. of } \frac{5}{2} = \text{hyp. log. of } 2 \cdot 5 = 0 \cdot 9162907,$$

$$1 + 0 \cdot 9162907 = 1 \cdot 9162907,$$

$$2 \times 1 \cdot 9162907 = 3 \cdot 8325814.$$

$$22 \text{ lbs.} = \text{load},$$

$$22 \times 5 = 110.$$

$$\frac{110}{3 \cdot 8325814} = 29 \text{ lbs. per square inch pressure nearly.}$$

Example 3.—The pressure of steam upon the piston is 40 lbs. per square inch; the resistance arising from imperfect condensation, 3 lbs. per square inch; the length of the stroke, 12 feet; and the steam is cut off at $\frac{1}{6}$ of the stroke: find the number of units of work done upon each square inch of the piston, and the number of units of work gained by working expansively. Also find the load per square inch, and the position of the piston when the velocity is greatest.

$40 \times 2 = 80 =$ work done before expansion begins.

$$\text{Hyp. log. of } \frac{12}{2} = \text{hyp. log. of } 6 = 1 \cdot 7917594,$$

$1 \cdot 7917594 \times 80 = 143 \cdot 340752$, units of work done after expansion begins.

80·00 units of work before expansion.
143·34 units of work after expansion.

223·34 whole work done on one square inch in one stroke.

But as the resistance from uncondensed vapour is 3 lbs. per square inch, for the whole length of the stroke, it will be 12 times 3; because 3 lbs. resisting through 12 feet gives 36 lbs. for the whole. We must subtract this from the whole work done to obtain the effective work.

223·34 = whole work done on one square inch.
36·00 = resistance of the uncondensed vapour on one square inch.

Diff. 187·34 = effective work on one square inch.

To find the advantage gained by working expansively; when the engine works without expansion, we have
$12 \times 40 = 480 =$ units of work done upon each square inch.

But, as in this case the steam is not cut off till the end of the stroke, there is $\frac{12}{2} = 6$ times the steam used; hence, for the same quantity, we have

$$\frac{480}{6} = 80 \text{ lbs. per square inch.}$$

But working expansively, we have obtained 223·34 units of work; hence $223·34 - 80 = 143·34$ lbs. gained in this example. Now, $\frac{223·34}{80} = 2·8$ times as much work done by the same quantity of steam when worked expansively.

To find the load. By rule 2.

$$\frac{223·34}{12} = 18·6 \text{ lbs., the load per square inch.}$$

To find where the velocity of the piston is greatest. By rule 4.

The hyp. log. of $\frac{12}{2} =$ hyp. log. of $6 = 1·7917594$
Add 1
the divisor mentioned in the rule. 2·7917594

Hence $\frac{12}{2·7917} = 4·3 =$ the number of feet travelled by the piston before acquiring its greatest volocity.

When it is stated that the steam is cut off at $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c., of the stroke, there is no necessity for dividing the distance moved by the piston before expansion by the length of the stroke, whatever may be the fractional part; we need only multiply the hyperbolic logarithm of the denominator by the work done before expansion begins, and the product is the work done by expansion: for example, suppose the length of the stroke to be 12 feet, and the steam is cut off at $\frac{1}{3}$ of the stroke, the pressure upon the piston being 60 lbs. per square inch.

Here the denominator is 3; its hyperbolic logarithm is 1·0986123.

The steam being cut off at 4 feet, the work done before expansion is $4 \times 60 = 240$; and $1·0986123 \times 240 = 263·6669520$ = the units of work done by expansion, and

263·6669520
240
―――――――
503·6669520 = the whole work done upon each square inch in one stroke.

The following rule is a good approximation.

Rule.—Divide that part of the stroke through which the expansion takes place, into any even number of equal parts, and calculate the pressure per square inch upon the piston at each division of the stroke; take the sum of the extreme pressure in pounds per square inch, four times the sum of the even pressures, and twice the sum of the odd pressures; multiply the sum of all these by one-third of the common distance between the positions of the piston, and the result will be the work done upon each square inch of the piston after expansion begins. The work done before the expansion begins is evidently equal to the pressure per square inch multiplied by the number of feet described before expansion. The whole work done during a single stroke is equal to the sum of the works done before and after expansion.

Example.—The pressure of steam upon the piston is

40 lbs. per square inch; the resistance arising from imperfect condensation, 3 lbs. per square inch; the length of the stroke, 12 feet; and the steam is cut off at $\frac{1}{6}$ of the stroke: find the number of units of work done upon each square inch of the piston.

The steam being cut off at 2 feet, divide the remaining part of the stroke, viz., 10 feet, into 10 equal parts.

Then, by Mariotte's law,

$$3 : 2 :: 40 : P_1.$$

$$P_1 = \frac{2 \times 40}{3} = 26{\cdot}666.$$

In the same way we have—

$$P_2 = \frac{2 \times 40}{4} = 20. \qquad P_6 = \frac{2 \times 40}{8} = 10.$$

$$P_3 = \frac{2 \times 40}{5} = 16. \qquad P_7 = \frac{2 \times 40}{9} = 8{\cdot}888.$$

$$P_4 = \frac{2 \times 40}{6} = 13{\cdot}333. \qquad P_8 = \frac{2 \times 40}{10} = 8.$$

$$P_5 = \frac{2 \times 40}{7} = 11{\cdot}428. \qquad P_9 = \frac{2 \times 40}{11} = 7{\cdot}273.$$

$$P_{10} = \frac{2 \times 40}{12} = 6{\cdot}666.$$

40 + 6·666 = 46·666 = sum of extreme pressures.

26·666
16·000
11·428
8·888
7·273

70·255 = sum of even pressures.
4

281·020 = four times the sum of even pressures.

$$
\begin{array}{r l}
20{\cdot}000 & \\
13{\cdot}333 & \\
10{\cdot}000 & \\
8{\cdot}000 & \\
\hline
51{\cdot}333 & = \text{sum of the odd pressures.} \\
2 & \\
\hline
102{\cdot}666 & = \text{twice the sum of the odd pressures.} \\
281{\cdot}020 & \\
46{\cdot}666 & \\
\hline
3)\,430{\cdot}352 & \\
\hline
143{\cdot}451 & = \text{work done by expansion.} \\
80{\cdot}000 & = \text{work done before expansion.} \\
\hline
223{\cdot}451 & = \text{whole work done upon each square inch.} \\
36{\cdot}000 & = \text{resistance from uncondensed vapour.} \\
\hline
187{\cdot}451 & = \text{whole effective work.}
\end{array}
$$

De Pambour, who has made a great number of experiments, estimates the resistances as follows:—

The pressure of the atmosphere about 15 lbs. per square inch; also the resistance arising from the various parts of the engine, at a mean, the estimate is about 1 lb. to the square inch for the unloaded engine, and an additional friction of ·14, or $\frac{1}{7}$ of the effective pressure or useful load, for overcoming the friction of the loaded engine.

Supposing the pressure of the steam is 60 lbs. per square inch; the resistances are 15 lbs. per square inch from the atmosphere, 1 lb. for the friction of the unloaded engine; then $15 + 1 = 16$, and this taken from 60, leaves 44; therefore the load $+ \frac{1}{7}$ of the load is equal to 44 lbs., or $\frac{8}{7}$ of the load $= 44$; therefore the load $= \frac{44 \times 7}{8} = 38\frac{1}{2}$, or 38·5. The load is, therefore, the effective pressure on the piston.

For high-pressure engines we have the

Load $+ \frac{1}{7}$ load $+ 16 =$ whole pressure of the steam.

Hence the following rule: *To find the load when friction is taken into account*, viz., THE USEFUL LOAD.

From the pressure of the steam in the cylinder, subtract 16; multiply the remainder by 7, and divide this product by 8, and the quotient is the useful load.

Example.—Given the pressure of steam in the cylinder 50 lbs. per square inch, to find the load.

Then, by the rule, $50 - 16 = 34$; then $\frac{34 \times 7}{8} = 29{\cdot}75$.

For the condensing or low-pressure engine, instead of the resistance of the atmosphere, we must use the resistance of the vapour in the condenser, which is generally estimated at about 4 lbs. to the square inch: in this case we have the

Load $+ \frac{1}{7}$ load $+ 1 + 4 =$ whole pressure of steam.

Hence we have the following rule:

From the mean pressure of the steam subtract 5; multiply the remainder by 7; this product divided by 8 gives the useful load.

On the Evaporating Power of the Boiler.

The evaporating power of the boiler is of the greatest importance, and as it is the source of all work in the steam engine, many ingenious contrivances have been made to increase this evaporating power; these various contrivances will hereafter be explained. The quantity of work done depends on the quantity of water evaporated, the temperature, and the pressure at which the steam is generated; we shall also hereafter give formulæ to find the relation between the volume of steam and the pressure; but the application of the following experimental table will give results sufficiently correct for all practical purposes. It shows the volume which a cubic foot of water has in the form of steam at the different pressures, as well as the corresponding temperatures.

Total pressure, in pounds, per square inch.	Corresponding temperature by Fahrenheit's thermometer.	Volume of the steam compared to the volume of the water that has produced it.	Total pressure, in pounds, per square inch.	Corresponding temperature by Fahrenheit's thermometer.	Volume of the steam compared to the volume of the water that has produced it.
1	102·9	20954	56	289·6	498
2	126·1	10907	57	290·7	490
3	141·0	7455	58	291·9	482
4	152·3	5695	59	293·0	474
5	161·4	4624	60	294·1	467
6	169·2	3901	61	294·9	460
7	176·0	3380	62	295·9	453
8	182·0	2985	63	297·0	447
9	187·4	2676	64	298·1	440
10	192·4	2427	65	299·1	434
11	197·0	2222	66	300·1	428
12	201·3	2050	67	301·2	422
13	205·3	1903	68	302·2	417
14	209·0	1777	69	303·2	411
15	213·0	1669	70	304·2	406
16	216·4	1572	71	305·1	401
17	219·6	1487	72	306·1	396
18	222·6	1410	73	307·1	391
19	225·6	1342	74	308·0	386
20	228·3	1280	75	308·9	381
21	231·0	1224	76	309·9	377
22	233·6	1172	77	310·8	372
23	236·1	1125	78	311·7	368
24	238·4	1082	79	312·6	364
25	240·7	1042	80	313·5	359
26	243·0	1005	81	314·3	355
27	245·1	971	82	315·2	351
28	247·2	939	83	316·1	348
29	249·2	909	84	316·9	344
30	251·2	882	85	317·8	340
31	253·1	855	86	318·6	337
32	255·0	831	87	319·4	333
33	256·8	808	88	320·3	330
34	258·6	786	89	321·1	326
35	260·3	765	90	321·9	323
36	262·0	746	91	322·7	320
37	263·7	727	92	323·5	317
38	265·3	710	93	324·3	313
39	266·9	693	94	325·0	310
40	268·4	677	95	325·8	307
41	269·9	662	96	326·6	305
42	271·4	647	97	327·3	302
43	272·9	634	98	328·1	299
44	274·3	620	99	328·8	296
45	275·7	608	100	329·6	293
46	277·1	596	105	333·2	281
47	278·4	584	120	343·3	249
48	279·7	573	135	352·4	224
49	281·0	562	150	360·8	203
50	282·3	552	165	368·5	187
51	283·6	542	180	375·6	173
52	284·8	532	195	382·3	161
53	286·0	523	210	388·6	150
54	287·2	514	225	394·6	141
55	288·4	506	240	400·2	133

Examples.

1. Given the area of the piston of a high-pressure engine 200 square inches, the length of the stroke 4 feet, the evaporation of the boiler half a cubic foot of water per minute, the pressure of steam in the cylinder 60 lbs. per square inch: find the useful load and the horse power.

To find the Useful Load.

By the rule (page 10), $60 - 16 = 44$; then $44 \times \frac{7}{8} = 38{\cdot}5$ lbs. = useful load.

To find the Volume of Steam evaporated per Minute.

Multiply the evaporation of the boiler in cubic feet per minute by the volume in the table corresponding to the given pressure, and the product is the volume of steam evaporated per minute.

Here the pressure is 60 lbs., the corresponding volume in the table is 470.

The evaporating power of the boiler being half a cubic foot per minute, we have

$470 \times \frac{1}{2} = 235$ for the number of cubic feet evaporated per minute.

The number of cubic feet discharged per stroke is equal to the area of the piston in feet, multiplied by the length of the stroke in feet $= \frac{200}{144} \times 4 = \frac{800}{144} = 5{\cdot}55$.

The whole volume discharged per minute is equal to the number of strokes per minute, multiplied by the volume discharged at one stroke; and the volume discharged per minute must be equal to the volume evaporated per minute.

Number of strokes per minute $\times$ 5·55, whole discharge in one minute = 235 lbs. evaporated also in one minute.

$$\text{Hence the number of strokes} = \frac{235}{5{\cdot}55} = 42.$$

But the useful work done in one stroke is

$$38{\cdot}5 \times 200 \times 4 = 30800,$$

therefore the useful work per minute is

$$38{\cdot}5 \times 200 \times 4 \times 42, \text{ or } 30800 \times 42;$$

therefore the horse power is $\dfrac{30800 \times 42}{33000} = 39.$

Example 2.—In a condensing engine the area of the cylinder is 1440 square inches; the length of the stroke, including clearance, is 5 feet; the steam is cut off at 1 foot; the clearance is $\frac{1}{4}$ of a foot; the pressure of steam is 30 lbs.; the elasticity of the vapour in the condenser is 4 lbs.; the effective evaporation of the boiler is ·2 of a cubic foot per minute, and the resistances as usual: required the useful load, and the useful horse power.—*Tate's Mechanics.*

The space through which the piston moves before the steam is cut off $= 1 - \dfrac{1}{4} = \dfrac{3}{4}$, and the whole length of the stroke is $5 - \dfrac{1}{4} = 4\frac{3}{4}$.

The work done before the steam is cut off

$$= 30 \times \frac{3}{4} = \frac{90}{4} = 22{\cdot}5.$$

To find the work done by expansion

$$4\tfrac{3}{4} = \frac{19}{4}; \text{ then } \frac{19}{4} \text{ divided by } \frac{3}{4} = \frac{19}{3} = 6{\cdot}33.$$

$$\text{Hyp. log. of } 6{\cdot}33 = 1{\cdot}8453002.$$

$1{\cdot}8453002 \times 22{\cdot}5 = 41{\cdot}5192545 =$ work done by expansion.

Hence the whole work in one stroke is

$$22{\cdot}5 + 41{\cdot}5192545 = 64{\cdot}0192545.$$

$$\text{Mean pressure} = \frac{64{\cdot}0192545}{4{\cdot}75} = 13{\cdot}48 \text{ lbs.}$$

To find the Useful Load.

By the rule, $13{\cdot}48 - 5 = 8{\cdot}48$; then $8{\cdot}48 \times \dfrac{7}{8} = 7.42$.

Now, one cubic foot of water, by the Table, expands into 882 cubic feet of steam at 30 lbs. pressure.

Hence the volume of steam evaporated per minute is

$$\cdot 2 \times 882 = 176\cdot 4 \text{ cubic feet.}$$

Volume of steam discharged in one stroke is

$$\frac{1440}{144} \times 1 = 10 \text{ cubic feet.}$$

Number of strokes per minute, multiplied by number of cubic feet discharged in one stroke, is equal to number of cubic feet discharged per minute, which must be equal to the cubic feet evaporated per minute.

Therefore the number of strokes per minute

$$= \frac{\text{number of cubic feet evaporated in one minute.}}{\text{number of cubic feet discharged in one stroke.}}$$

$$\text{Number of strokes} = \frac{176\cdot 4}{10} = 17\cdot 64.$$

The useful work per minute is found by multiplying together the area of the piston, the useful load, the length of the stroke, and the number of strokes per minute.

$$\text{Horse power} = \frac{1440 \times 7\cdot 42 \times 4\cdot 75 \times 17\cdot 64}{33000} = 27\cdot 13.$$

On the Duty of the Engine.

Mr. Pole, who is one of the ablest cultivators of practical science in this country, when speaking of the great duty of the Cornish engines, observes: "Another cause of the great duty is the *use of high-pressure steam.* This is not only advantageous indirectly, inasmuch as it enables the expansive principle to be applied with greater effect, but there is a totally distinct economical advantage in the use of high-pressure steam *per se*, which is often overlooked, and deserves special mention. It is founded on the principle, *that the pressure of the steam increases in a greater ratio than its density;* whence it follows that

the higher pressure the steam is raised to, the less *proportionate* quantity of water it contains, and therefore the less fuel is consumed, since a given quantity of fuel will evaporate the same weight of water at all temperatures."

We shall take the example which Mr. Pole gives at page 169 of his work on the Cornish engine, and solve it by the rules we have given.

Example.—A Cornish engine, with a cylinder 70 inches in diameter, and a 10-feet stroke, has the steam admitted at a pressure of 45 lbs. to the square inch (*i. e.* 30 lbs. above the atmosphere) during $\frac{1}{6}$ of the stroke, and during the remainder the steam is allowed to expand.

We will ascertain, laying aside the minutiæ of the question, what duty this engine ought to perform for a bushel of coals consumed. The diameter of the cylinder being 70 inches, the area is 3848 square inches; this multiplied by 45 lbs., the pressure on each square inch, gives the whole pressure on the piston $= 173160$ lbs. This is moved through $1\frac{2}{3}$ feet, the space during which the steam is admitted; hence a quantity of work $= 173160 \times 1\frac{2}{3} = 288600$ lbs. raised one foot high before the steam is cut off.

To calculate the Work done by Expansion.

By rule 1. As the steam is cut off at $\frac{1}{6}$ of the stroke, we must take the hyperbolic logarithm of 6, which is 1·79176; then

$$1{\cdot}79176 \times 288600 = 517102.$$

Hence the work done by the steam in the whole stroke is $288600 + 517102 = 805702$ lbs. raised one foot high.

The area of the cylinder being 3848 square inches, and the steam being admitted during 20 inches of the stroke, the quantity of steam consumed to do the above work will be 76960 cubic inches; and since the relative densities of water and steam at 45 lbs. pressure are 1 and 608 (see

table, p. 11), this volume of steam will contain $\frac{76960}{608} =$ 126 cubic inches of water; but, as each cubic foot weighs 62·5 lbs., and 1728 cubic inches = one cubic foot, we have—

$$
\begin{array}{r}
1728 : 126 :: 62{\cdot}5 \\
126 \\
\hline
375{\cdot}0 \\
1250 \\
625 \\
\hline
1728)\,7875{\cdot}0\,(4{\cdot}55 \\
6912 \\
\hline
9630 \\
8640 \\
\hline
990
\end{array}
$$

that is, 128 cubic inches of water weigh 4·55 lbs.

On the supposition that 1 lb. of coal will evaporate 9·271 lbs. of water, we can show the quantity of coal required for each stroke, thus: 9·271 lbs. of water must bear the same proportion to 4·55 lbs. of water, as one pound of coal bears to the pounds of coal required for each stroke,

$$
\begin{array}{r}
9{\cdot}271 : 4{\cdot}55 :: 1 \\
1 \\
\hline
9{\cdot}271)\,4{\cdot}5500\,({\cdot}49 \\
37084 \\
\hline
84160 \\
83439 \\
\hline
721
\end{array}
$$

That is, ·49 lbs., which is nearly half a pound of coal, is required for each stroke, to produce a motive power of 805702 lbs. raised one foot high, at which rate one bushel (94 lbs.) of coal will give an amount of upwards of 154,000,000, for

$$
\begin{array}{r}
\cdot 49 : 94 :: 805702 : \\
94 \\
\hline
3222808 \\
7251318 \\
\hline
\cdot 49)\ 75735988\ (154563445 \\
\hline
\end{array}
$$

Taking it in round numbers to be 154,000,000, and estimating the net duty at 80,000,000, which may be considered a good duty for such a degree of expansion, we have 74,000,000, which is equivalent to about 10 lbs. per square inch, to allow for friction, imperfect vacuum, loss by the steam jackets, cooling, &c.: with a greater expansion, greater effects will be produced.

To find the point at which the steam must be cut off to obtain the greatest quantity of useful work.

It is clear that there is some part of the stroke where the steam ought to be cut off which is preferable to any other; for, if the pressure of the steam at the end of the stroke be greater than the useless resistances, all the work is not taken out of the steam, or, in other words, the steam could have done more useful work; but if, on the contrary, the pressure of the steam at the end of the stroke be less than those resistances, then it is also clear that the steam has been cut off too soon. Therefore, it is evident that the steam should be cut off at a point such, that the pressure be just equal to all the above-named useless resistances and no more.

*Rule**.—Divide 24250 by the pressure per square inch on the piston, to the quotient add 65 for a dividend.

* This rule is made from the formula given by Pole.

Divide 24250 by the useless resistances, and to this quotient add 65 for a divisor.

The quotient arising from dividing the former by the latter, multiplied by the length of the stroke, will give the part of the stroke where the steam must be cut off to obtain the greatest amount of useful work.

Example.—Suppose the length of the stroke of the piston of a steam engine to be 10 feet; the pressure 40 lbs. per square inch: required at what part of the stroke the steam must be cut off in order to obtain the greatest amount of useful work, when the ***vacuum resistance, together with the friction of the engine**** = 5 lbs.

Then, by the rule, $\frac{24250}{40} = 606{\cdot}25$

$$606{\cdot}25 + 65 = 671{\cdot}25$$

$$\frac{24250}{5} = 4850$$

$$4850 + 65 = 4915$$

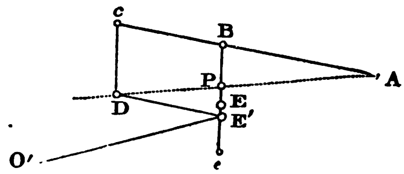
$$\frac{671{\cdot}25}{4915} = 0{\cdot}136.$$

$0{\cdot}136 \times 10 = 1{\cdot}36$ feet, the part of the stroke where the steam must be cut off to obtain the greatest amount of useful work.

Parallel Motion.

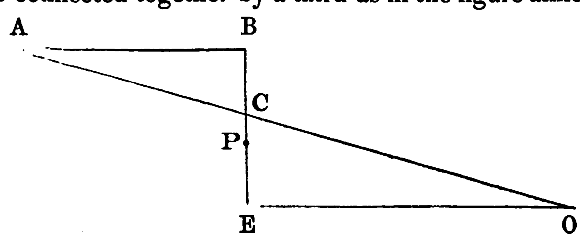
There are few parts of the steam engine less understood by the mere practical man than parallel motion, it requiring a pretty considerable knowledge of geometry to enter into it fully; neither have all theoretical writers had clear views of this subject.

* Pambour considers the resistance from imperfect condensation to be 4 lbs. per square inch, and the friction of the unloaded engine 0·5 lbs.



Professor Millington, for instance, in his *Epitome of Natural Philosophy,* says, that the length of the radius rod E′O′ is always equal to AB, which is only true when AB = BC; but this is not the only mistake the Professor makes, for he says that the point E′ describes a right-lined motion, which it communicates or transmits to D. Now this is so clearly a mistake, that it scarcely need be pointed out, since, if E′ be movable round O′ as a centre, it will evidently describe a circle; but there is a point in B E′ which will describe a line that will not differ sensibly from a right line, but which in reality is a curve line resembling the figure 8.

Mr. Watt, who was the inventor of parallel motion, describes it in *Robinson's Mechanical Philosophy,* as two bars connected together by a third as in the figure annexed.



AB being one beam or bar and EO the other, generally called the radius rod, connected together by the bar or link BE.

For Lever Engines.

Having given the length of the levers and that of the link which connects them, to find at what point of the link the piston rod is to be attached so as to move in a vertical line nearly.

If the levers be of equal length, the piston rod must be attached to the middle of the link.

If the levers be unequal, draw a line from the centre of motion of the one lever to that of the other; this will cut the link in some point; take the distance of this point from the top of the link, and set it off from the bottom of the link; the point thus obtained is that to which the piston rod must be attached.—This practical method may be easily understood by persons that are unacquainted even with the processes of common arithmetic.

Rule.—Multiply the length of the beam from the centre of motion by the length of the link for a dividend.

Add the length of the beam to that of the radius rod for a divisor.

The quotient will be the distance from the bottom of the link to which the piston rod must be attached.

Example.—Given the length of the beam or lever A B = 6 feet; the length of the radius rod E O = 9 feet; the length of the link B E = 5 feet: find how far from the bottom of the link the piston rod must be put on.

$$6 \times 5 = 30 = \text{dividend.}$$
$$6 + 9 = 15 = \text{divisor.}$$

$\frac{30}{15} = 2 =$ the quotient $=$ the distance E P from the bottom of the link at which the piston rod is to be attached.

If the upper lever be given, and the point in the link where the piston rod is attached, to find the length of the lower lever, which is generally called the radius rod.

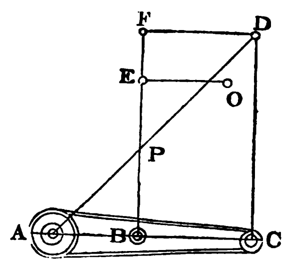
Rule.—As the distance of the point from the bottom is to that from the top of the link, so is the length of the upper lever to the length of the radius rod.

Example.—Given the length of the lever A B = 6 feet; the length of the link B E = 5 feet; and the distance E P from the bottom of the link at which the piston rod is attached = 2 feet: find the length E O of the radius rod.

As the whole length of the link is 5 feet, and E P is equal to 2 feet, the remaining part B P of the link must be equal to 3 feet. Then, by the rule,

$$2 : 3 :: 6 : \text{radius rod}.$$

$$\text{Hence the radius rod} = \frac{6 \times 3}{2} = 9 \text{ feet}.$$



This proposition is very simply applied to find the length of the radius rods for a steam-boat parallel motion; we have only to join A, the centre of motion of the beam, and D, the centre of the cross-head, by the line A D, cutting B E in P; then, since AC : CD :: AB : BP, an invariable ratio, whatever kind of a line the point D describes, the very same kind of line will the point P describe, but shorter in the proportion of A B : A C; therefore, all that is necessary is to make P move in a line approximating to a vertical straight line, and the point D will move in the line as if the calculation had been made for D.

It is, therefore, of great importance to know that the point P describes the same kind of line as the point D, for this knowledge enables us to abstract entirely from the point D, and apply our calculations so as to make the point P move in a vertical straight line nearly. It is quite clear that the apparent complication of rods is hereby reduced to the simplest case of parallel motion, viz., that of two bars connected by a link, A B being one bar, E O (which is called the radius rod) the other, and E B the link connecting them. Therefore, we have only to stretch a line from A to D, and the point where this line cuts the back link E B, is that which must be made to describe a right line nearly, or by calculation,

AC : CD :: AB : BP.

Hence P is known.

As an example, given the radius of the beam $AC = 10$ feet; length of the side rod $CD = 12$ feet; and length of back link $EB = 11$ feet: to find the length of the radius rod EO; AB being 5 feet.

$$\text{As } AC : CD :: AB : BP, \text{ that is,}$$
$$10 : 12 :: 5 : 6.$$
$$\begin{array}{r} 5 \\ \hline 10)\,60 \\ \hline 6 \text{ feet;} \end{array}$$

that is, $BP = 6$ feet, and since $BE = 11$ feet, $EP = 5$ feet; then as

$$EP : BP :: AB : EO, \text{ that is,}$$
$$5 : 6 :: 5 : 6$$
$$\begin{array}{r} 5 \\ \hline 5)\,30 \\ \hline 6 \text{ feet, length of radius rod.} \end{array}$$

For full Parallel Motion.

Rule.—Divide the square of the distance of the back link from the centre of motion by the length of the parallel bar, and the quotient will be the length of the radius rod.

Example.—Given the radius of the beam, 12 feet, and the length of the parallel bar, 5 feet: to find the length of the radius rod.

As the length of the parallel bar is 5 feet, the distance of the back link from the centre of motion will be $12 - 5 = 7$ feet.

Hence, by the rule, $\frac{7^2}{5} = \frac{49}{5} = 9\frac{4}{5}$ feet, the length of the radius rod.

The radius rod being made to work from a stated centre, find the length of the parallel bar and that of the radius rod.

Rule.—Add the distance between the vertical line and the axis of the radius rod to twice the radius of the beam.

Divide the square of the radius of the beam by the above sum, and the quotient will be the length of the parallel bar.

If the radius rod be shorter than the parallel bar, divide the square of the radius of the beam by the difference instead of the sum.

The distance between the vertical line and the axis of the radius rod, added to or subtracted from the parallel bar, according as the radius rod is longer or shorter than the parallel bar, gives the length of the radius rod.

Or, add the radius of the beam to the distance between the vertical line and axis of the radius rod; divide the square of this sum by twice the radius of the beam added to the above-named distance for the length of the radius rod.

When the length of stroke is taken into consideration, subtract the square of half the length of the stroke from the square of the radius of the beam, and extract the square root of the remainder; to this root add the above radius of the beam, and multiply this sum by half the radius of the beam, for a dividend.

Then add the above root, the radius of the beam, and the distance between the vertical line and the end of the radius rod together, for a divisor.

And if the above dividend be divided by this divisor, the quotient will give the length of the parallel bar; and this, added to the distance between the vertical line and the end of the radius rod, will give the length of the radius rod.

Example.—Given the length of the beam, 12 feet; the length of the stroke, 6 feet; and the distance between the vertical line and the end of the radius rod $=$ 4 feet: to find the length of the radius rod.

By the rule, $12^2 = 144$ and $3^2 = 9$; and $144 - 9 = 135$, the square root of which is 11·62 nearly.

Then $11 \cdot 62 + 12 = 23 \cdot 62$; and $23 \cdot 62 \times 6 = 141 \cdot 72$, which is the dividend mentioned in the rule.

Next, $11 \cdot 62 + 12 + 4 = 27 \cdot 62$, the divisor; therefore 141·72 divided by 27·62 gives 5·13, the length of the parallel bar; consequently $5 \cdot 13 + 4 = 9 \cdot 13$ feet, length of radius rod.

In the above example, when the length of the stroke is neglected, the radius of the beam = 12 feet; and the distance between the vertical line and the end of the radius rod = 4 feet.

Then, by the rule, we have, $12 + 4 = 16$,
and $2 \times 12 + 4 = 28$.

Hence $\frac{16 \times 16}{28} = \frac{256}{28} = 9 \cdot 14$, which only differs 0·01 from the above result. If the radius rod be shorter than the parallel bar, it is only necessary to use subtraction instead of addition in the first rule. Thus, suppose that the axis of the radius rod had been 4 feet on the other side of the vertical line, then $12 - 4 = 8$, and $12 \times 2 - 4 = 24 - 4 = 20$.

Hence $\frac{8 \times 8}{20} = \frac{64}{20} = 3\frac{1}{5}$ = length of the radius rod.

Examples in Steam-boat Parallel Motion.

Example 1.—Given the radius of the beam AC = 5 feet; the length of the link or side rod CD = 4 feet 2 inches; and the distance AB of the back link from the centre of motion, 3 feet: it is required to determine the length of the radius rod, when it is attached to a point P, 4 feet above B.

Now, AC : CD : : AB : BP,
That is, 60 : 50 : : 36 : 30.

$$\begin{array}{r} 36 \\ \hline 60)\,1800 \\ \hline 30 = \text{BP}; \end{array}$$

$$BE - BP = EP = 48 - 30 = 18,$$

$$EP : PB :: AB : OE,$$

$$18 : 30 :: 36 : 60$$

$$\begin{array}{r} 36 \\ \hline 18)1080 \\ \hline \end{array}$$

60 inches = 5 feet = OE, the length of the radius rod required.

Example 2.—Let the radius of the beam AC = 5 feet, as in the last example, and AB = 2 feet, but the length of the radius rod confined be only 8 inches: it is required to determine the distance of the point E from B, where the radius rod must be attached, the length of the links being the same as in the last example.

$$AC : CD :: AB : BP$$

$$60 : 50 :: 24$$

$$\begin{array}{r} 50 \\ \hline 6{,}0)\,120{,}0 \\ \hline 20 = BP \end{array}$$

$$OE : AB :: BP : EP$$

$$8 : 24 :: 20$$

$$\begin{array}{r} 20 \\ \hline 8)\,480 \\ \hline 60 = 5 \text{ feet} = EP; \end{array}$$

and 60 + 20 = 80 = 6 feet 8 inches = BE, the length of the back link.

Practical Observations.

Since we have given rules for finding the length of radius rods, it now becomes necessary to show how to put them on.

Plumb the piston rod when the piston is at the top extremity of its stroke; then, one end of the radius rod

being movable about E′, with the other end o′ describe an arc of a circle. Now, bring the piston down to the lowest extremity of its stroke, and again plumb the piston rod, and in the same manner as before describe another arc of a circle; and the point where these arcs intersect is the centre upon which the end of the radius rod is to move.

It is a practice among engineers to set the cylinder half the vibration of the beam in towards the centre of the beam; and we shall here show how to find the vibration or versed sine of the arc described.

Rule.—From the square of the length of the beam, taken from the centre of motion, subtract the square of half the length of the stroke; and the square root of the remainder, subtracted from the above length of the beam, will give the vibration required.

Example.—Given the length of the beam from the centre of motion, 5 feet; and half the length of the stroke, 3 feet: to find the vibration.

Now, $5^2 = 25$, and $3^2 = 9$; then, $\sqrt{(25 - 9)} = \sqrt{16} = 4$. $5 - 4 = 1$. That is, the vibration will be one foot.

Therefore, in this case, the horizontal distance between the centre of motion and the centre of the cylinder must be 4 feet 6 inches.

When an engine works with a vibrating pillar, the vibration is in an opposite direction, and the centre of the vibrating pillar axle must be set half the vibration in towards the cylinder.

The air-pump bucket rod must he hung on at the point F; for the point F describes the same kind of line as the piston rod describes.

To find the Length of the Connecting Rod.

Set the beam at half stroke, that is, parallel to the horizon; and the distance between the centre of the pin on which the connecting rod is to move and the centre of the shaft, is the length of the connecting rod.

Some practical men set both the beam and crank parallel

to the horizon, and take the distance between the centre of the pin on which the connecting rod is to move, and the centre of the crank pin, for the length of the connecting rod.

This method has also been given by some writers; but it is incorrect, and when the length is so taken, the connecting rod will be found too long.

Rules and Examples for the Fly-wheel.

For the double-acting engine, the number of revolutions, or the number of double strokes per minute, the mean radius, and the horse power being given.

To find the Weight of the Wheel.

Rule 1.—Multiply the number of horse power by 2275, and that product by n*. Multiply the square of the mean radius by the cube of the number of revolutions per minute.

Divide the former product by the latter, and the quotient will be the weight in tons.

Example.—A double-acting engine makes 15 revolutions per minute; the radius of the fly-wheel is 15 feet; the horse power is 60: what must be the weight of the fly-wheel, supposing the variation to be $\frac{1}{40}$ from the mean velocity?

In this case $n = 40$; then, by the rule,

$$60 \times 2275 \times 40 = 5460000,$$
$$15^2 \times 15^3 = 225 \times 3375 = 759375,$$
$$\frac{5460000}{759375} = 7{\cdot}2 \text{ tons nearly.}$$

* Morin, at page 191 of the *Aide Mémoire de Mécanique Pratique*, says that "(n) should be taken $= 20$ or 25 for engines which are not required to work with great velocity, such as flour mills, saw mills, &c. For engines working spinning or weaving machines, (n) should be taken 35 or 40. When the spinning is to be done with very great regularity, (n) should be 50 or 60."

To find the mean Radius of the Wheel.

Rule 2.—Multiply the number of horse power by n, divide the product by the area of the section of the rim, and extract the cube root of the quotient.

Divide 12·17 by the number of revolutions per minute, and multiply the quotient by the cube root before obtained. The product will be the mean radius required.

Example.—A double-acting engine makes 10 revolutions per minute; the horse power is 60; the area of the section of the rim is 1·3 feet: supposing the variation to be $\frac{1}{40}$ from the mean velocity, what must be the mean radius of the wheel?

In this case $n = 40$; then, by the rule,

$$60 \times 40 = 2400$$

$$\frac{2400}{1 \cdot 3} = 1846 \cdot 15$$

$$\sqrt[3]{1846 \cdot 15} = 12 \cdot 27, \text{ and } \frac{12 \cdot 17}{10} = 1 \cdot 217$$

$1 \cdot 217 \times 12 \cdot 27 = 14 \cdot 93$ feet = mean radius of the wheel.

To find the Area of the Section of the Rim.

Rule 3.—Multiply 1802·9 by the number of horse power, and that product by n. Multiply the cube of the mean radius by the cube of the number of revolutions per minute.

Divide the former product by the latter, and the quotient will be the area of the section of the rim.

Example.—A double-acting engine makes 10 revolutions per minute; the horse power is 60; the mean radius of the wheel is 12 feet: supposing the variation to be $\frac{1}{20}$ from the mean velocity, what must be the section of the rim of the wheel?

In this case $n = 20$; then, by the rule,

$$1802 \cdot 9 \times 60 \times 20 = 2163480,$$

$$12^3 \times 10^3 = 1728 \times 1000 = 1728000,$$

$$\frac{2163480}{1728000} = 1 \cdot 25 \text{ feet.}$$

For the double-acting engine the number of *single strokes** per minute, the mean radius, and the horse power being given.

To find the Weight of the Wheel.

Rule 4.—Multiply the horse power by 18200, and that product by *n*. Multiply the square of the mean radius by the cube of the number of single strokes per minute.

Divide the former product by the latter, and the quotient will be the weight in tons.

Example.— A double-acting engine makes 30 single strokes per minute; the horse power is 40; the mean radius of the wheel is 10 feet: supposing the variation to be $\frac{1}{35}$ from the mean velocity, what must be the weight of the fly-wheel?

In this case $n = 35$; then, by the rule,

$$40 \times 18200 \times 35 = 25480000,$$

$$10^2 \times 30^3 = 100 \times 27000 = 2700000,$$

$$\frac{25480000}{2700000} = 9{\cdot}44 \text{ tons.}$$

To find the mean Radius of the Wheel.

Rule 5.—Multiply the number of horse power by *n*; divide the product by the area of the section of the rim, and extract the cube root of the quotient.

Divide 24·34 by the number of single strokes per minute, and multiply the quotient by the cube root before obtained. The product will be the mean radius required.

Example.—A double-acting engine of 50-horse power makes 25 single strokes per minute; the area of the section of the rim of the wheel is 1 square foot: supposing the variation to be $\frac{1}{20}$ from the mean velocity, what must be the mean radius of the wheel?

* The engine makes two *single strokes* for each *revolution* of the fly-wheel.

In this case $n = 20$; then, by the rule,

$$50 \times 20 = 1000$$

$$\frac{1000}{1} = 1000$$

$$\sqrt[3]{1000} = 10$$

$$\frac{24{\cdot}34}{25} = 0{\cdot}974$$

$$0{\cdot}974 \times 10 = 9{\cdot}74 \text{ feet.}$$

To find the Area of the Section of the Rim.

Rule 6.—Multiply 14423 by the number of horse power, and that product by n. Multiply the cube of the mean radius by the cube of the number of single strokes per minute.

Divide the former product by the latter, and the quotient will be the area of the section of the rim.

Example.—A double-acting engine of 40-horse power makes 20 single strokes per minute; the mean radius of the wheel is 14 feet: supposing the variation to be $\frac{1}{50}$ from the mean velocity, what must be the area of the section of the rim?

In this case $n = 50$; then, by the rule,

$$14423 \times 40 \times 50 = 28846000,$$

$$14^3 \times 20^3 = 2744 \times 8000 = 21952000,$$

$$\frac{28846000}{21952000} = 1{\cdot}3 \text{ square feet nearly.}$$

For the single-acting engine, the number of revolutions or the number of double strokes per minute, the mean radius, and the horse power being given.

To find the Weight of the Wheel.

Rule 7.—Multiply the number of horse power by 11860, and that product by n. Multiply the square of the mean radius by the cube of the number of revolutions per minute.

Divide the former product by the latter, and the quotient will be the weight in tons.

Example.—A single-acting engine of 30-horse power makes 20 revolutions per minute; the mean radius of the wheel is 12 feet: supposing the variation to be $\frac{1}{25}$ from the mean velocity, what must be the weight of the wheel?

In this case $n = 25$; then, by the rule,

$$11860 \times 30 \times 25 = 8895000,$$
$$12^2 \times 20^3 = 144 \times 8000 = 1152000,$$
$$\frac{8895000}{1152000} = 7{\cdot}72 \text{ tons, weight of wheel.}$$

To find the mean Radius of the Wheel.

Rule 8.—Multiply the number of horse power by n; divide the product by the area of the section of the rim, and extract the cube root of the quotient.

Divide 21·1 by the number of revolutions per minute, and multiply this quotient by the cube root before obtained. The product will be the mean radius required.

Example.—A single-acting engine of 50-horse power makes 15 revolutions per minute; the area of the section of the rim is 1·2 square feet: supposing the variation to be $\frac{1}{30}$ from the mean velocity, what must be the mean radius of the wheel?

In this case $n = 30$; then, by the rule,

$$50 \times 30 = 1500$$
$$\frac{1500}{1{\cdot}2} = 1250$$
$$\sqrt[3]{1250} = 10{\cdot}77$$
$$\frac{21{\cdot}1}{15} = 1{\cdot}41$$
$$1{\cdot}41 \times 10{\cdot}77 = 15{\cdot}19 \text{ feet, mean radius of the wheel.}$$

To find the Area of the Section of the Rim.

Rule 9.—Multiply 9395·8 by the number of horse power, and that product by n. Multiply the cube of the mean radius by the cube of the number of revolutions per minute.

Divide the former product by the latter, and the quotient will be the area of the section of the rim.

Example.—A single-acting engine of 20-horse power makes 14 revolutions per minute; the mean radius of the wheel is 9 feet: supposing the variation to be $\frac{1}{20}$ from the mean velocity, what must be the area of the section of the wheel?

In this case $n = 20$; then, by the rule,

$$9395{\cdot}8 \times 20 \times 20 = 3758320,$$

$$9^2 \times 14^3 = 729 \times 2744 = 2000376,$$

$$\frac{3758320}{2000376} = 1{\cdot}87 \text{ square feet.}$$

For the single-acting engine, the number of single strokes per minute, the mean radius, and the horse power being given.

To find the Weight of the Wheel.

Rule 10.—Multiply the number of horse power by 94880, and that product by n. Multiply the square of the mean radius by the cube of the number of single strokes per minute.

Divide the former product by the latter, and the quotient will be the weight of the wheel in tons.

Example.—A single-acting engine of 20-horse power makes 30 single strokes per minute; the mean radius of the wheel is 13 feet: supposing the variation to be $\frac{1}{25}$ from the mean velocity, what must be the weight of the fly-wheel?

In this case $n = 25$; then, by the rule,

$$94880 \times 20 \times 25 = 47440000,$$

$$13^2 \times 30^3 = 169 \times 27000 = 4563000,$$

$$\frac{47440000}{4563000} = 10{\cdot}4 \text{ tons nearly, weight of wheel.}$$

To find the mean Radius of the Wheel.

Rule 11.—Multiply the number of horse power by n;

divide the product by the area of the section of the rim, and extract the cube root of the quotient.

Divide 42·2 by the number of single strokes per minute, and multiply the quotient by the cube root before obtained. The product will be the mean radius required.

Example.—A single-acting engine of 60-horse power makes 40 single strokes per minute; the area of the section of the rim of the fly-wheel is 1·2 square foot; supposing the variation to be $\frac{1}{60}$ from the mean velocity, what must be the mean radius of the wheel?

In this case $n = 60$; then, by the rule,

$$60 \times 60 = 3600$$

$$\frac{3600}{1 \cdot 2} = 3000$$

$$\sqrt[3]{3000} = 14 \cdot 4$$

$$\frac{42 \cdot 2}{40} = 1 \cdot 055$$

$1 \cdot 055 \times 14 \cdot 4 = 15 \cdot 19$ feet, mean radius of the wheel.

To find the Area of the Section of the Rim.

Rule 12.—Multiply 75166 by the number of horse power, and that product by n. Multiply the cube of the mean radius by the cube of the number of single strokes per minute.

Divide the former product by the latter, and the quotient will be the area of the section of the rim.

Example.—A single-acting engine of 30-horse power makes 32 single strokes per minute; the mean radius of the wheel is 11 feet; supposing the variation to be $\frac{1}{30}$ from the mean velocity, what must be the area of the section of the wheel?

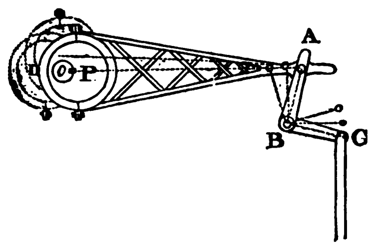
In this case $n = 30$; then, by the rule,

$$75166 \times 30 \times 30 = 67649400$$

$$11^3 \times 32^3 = 1331 \times 32768 = 43614208$$

$$\frac{67649400}{43614208} = 1 \cdot 55 \text{ square foot.}$$

To Construct an Eccentric Wheel.



From the centre of the shaft O take OP, equal to half the length of the *throw* which you intend the wheel to work; and from P as a centre, with any radius greater than PD, describe a circle, and this circle will represent the required wheel. For, every circle drawn from the centre P will work the same length of stroke, whatever may be its radius; as, whatever you increase the distance of the circumference of the circle from the centre of motion on the one side, you will have a corresponding increase on the opposite side equal to it.

Thus, suppose an eccentric wheel to work a stroke of 18 inches is required, the diameter of the shaft being 6 inches; and if 2 inches be the thickness of metal necessary for keying it on to the shaft, then set off, from O to P, 9 inches; and $9 + 5 = 14$ inches, the radius of the wheel required.

Rules for finding the Length of the Slide and Eccentric Levers, and the Throw of the Slide and Eccentric.

The *throw* of an eccentric is double the *eccentricity,* or equal to twice the distance between the centre of the eccentric wheel and the centre of the shaft.

To find the Length of the Eccentric Lever.

Rule 1.—Multiply the length of the slide lever by the throw of the eccentric, divide this product by the throw of the slide, and the quotient will be the length of the eccentric lever.

Example.—Given the throw of the slide $= 8$ inches, the length of the slide lever $= 4$ inches, and the throw of the eccentric $= 20$ inches: required the length of the eccentric lever.

$$\text{By the rule, } 4 \times 20 = 80$$
$$\frac{80}{8} = 10 \text{ inches.}$$

To find the Length of the Slide Lever.

Rule 2.—Multiply the length of the eccentric lever by the throw of the slide, divide this product by the throw of the eccentric, and the quotient will be the length of the slide lever.

Example.—Given the length of the eccentric lever = 10 inches, the throw of the eccentric = 20 inches, and that of the slide = 8 inches: required the length of the slide lever.

$$\text{By the rule, } 10 \times 8 = 80$$
$$\frac{80}{20} = 4 \text{ inches.}$$

To find the Throw of the Eccentric.

Rule 3.—Multiply the length of the eccentric lever by the throw of the slide, divide this product by the length of the slide lever, and the quotient will be the throw of the eccentric.

Example.—Given the throw of the slide = 4 inches, the length of the eccentric lever = 9 inches, and that of the slide lever = 3 inches: required the throw of the eccentric.

$$\text{By the rule, } 9 \times 4 = 36$$
$$\frac{36}{3} = 12 \text{ inches.}$$

To find the Throw of the Slide.

Rule 4.—Multiply the length of the slide lever by the throw of the eccentric, divide this product by the length of the eccentric lever, and the quotient will be the throw of the slide.

Example.—Given the length of the slide lever = 3 inches, that of the eccentric lever = 9 inches, and the throw of the eccentric = 12 inches: what must be the throw of the slide?

$$\text{By the rule, } 3 \times 12 = 36$$

$$\frac{36}{9} = 4 \text{ inches.}$$

On the Safety Valve.

The apertures for the safety valve require no nice calculation. It is only necessary to have the aperture sufficient to let the steam off from the boiler as fast as it is generated, when the engine is not at work.

In the Appendix to Tredgold on the Steam Engine, the following rule is given to find the area of the aperture:—

Rule.—Divide the area of the fire surface by the excess of the pressure above the atmosphere expressed in pounds per square inch, and the quotient will be the square of the diameter of the narrowest part of the valve in inches.

Example.—Required the aperture of a safety valve for a boiler with 60 square feet fire surface, pressure 5 lbs. per square inch above the atmosphere.

$$\text{Then, by the rule, } \frac{60}{5} = 12$$

$$\sqrt{12} = 3{\cdot}46 \text{ inches.}$$

This is the narrowest part of the aperture of the valve; it is better, however, to make it rather larger than smaller.

The safety valve is loaded sometimes by putting a heavy weight upon it, and sometimes by means of a lever with a weight to move along to suit the required pressure.

When the whole weight is put on the valve, to find the pressure to each square inch:—

Multiply the square of the diameter of the valve by ·7854, and this product will give the area, or number of square inches in the valve.

And if the whole weight upon the valve, in pounds, be divided by the number of square inches in the valve, the quotient will give the number of pounds pressure to each square inch on the valve.

Example.—If a weight of 40 lbs. be placed upon a

valve, the diameter of which is 3 inches, what will be the pressure to each square inch?

$$3^2 \times \cdot 7854 = 9 \times \cdot 7854 \doteq 7 \text{ square inches;}$$

$$\text{Then } \frac{40}{7} = 5\tfrac{5}{7} \text{ lbs. per square inch.}$$

On the Safety-Valve Lever.

Without taking into consideration the Weight of the Lever.

This being a lever of the third order, it may be easily calculated. It is well known, and indeed is almost self-evident, that if we have two, three, or four times, &c., the leverage, we shall have two, three, or four times, &c., the effect produced respectively, the weight remaining the same; hence the following rules.

To find the Weight per square inch on the Valve.

Rule 1.—Divide the length of the lever by the distance between the fulcrum and valve, and the quotient gives the leverage; and the leverage, multiplied by the weight, gives the whole weight upon the valve; and this product, divided by the number of square inches in the valve, gives the weight per square inch on the valve.

Example.—Given the whole length of the lever = 18 inches, the distance between the fulcrum and valve = 3 inches, the diameter of the valve = 2 inches, and the weight at the end of the lever = 20 lbs.: required the pressure per square inch on the valve.

Then, by the rule, $\frac{18}{3} = 6 =$ leverage.

$6 \times 20 = 120$ lbs. = whole weight upon the valve.
$\cdot 7854 \times 2^2 = \cdot 7854 \times 4 = 3\cdot 1416$ square inches, area of the valve.

$$\frac{120}{3\cdot 1416} = 38\cdot 2 \text{ lbs. per square inch.}$$

To find the Weight.

Rule 2.—Multiply the number of pounds per square inch by the number of square inches, and this product gives

the whole weight upon the valve, which divided by the leverage gives the weight.

Example. — Given the whole length of the lever = 21 inches, the distance between the fulcrum and valve = 3 inches, the pressure per square inch on the valve = 30 lbs., the area of the valve = 7 square inches: required the weight.

Then, by the rule, $30 \times 7 = 210$ lbs., whole weight on the valve.

$$\frac{21}{3} = 7 = \text{leverage.}$$

$$\frac{210}{7} = 30 \text{ lbs.} = \text{weight required.}$$

To find the Length of the Lever.

Rule 3.—Divide the whole weight upon the valve by the weight, and the quotient will give the leverage; and the leverage multiplied by the distance between the fulcrum and the valve gives the length of the lever.

Example.—Given the distance between the fulcrum and the valve = 4 inches, the total pressure upon the valve = 125 lbs., the weight = 25 lbs.: required the length of the lever.

Then, by the rule, $\frac{125}{25} = 5 =$ leverage.

$$5 \times 4 = 20 = \text{length of the lever.}$$

Example.—Given the whole length of the lever, 24 inches; the distance between the fulcrum and valve, 3 inches; the diameter of the valve, $2\frac{1}{2}$ inches: required the weight put on at the end of the lever, so as to have 50 lbs. per square inch upon the valve; also to divide the lever so as to have 40, 30, 20 lbs., &c., upon the valve with the same weight.

$(2{\cdot}5)^2 \times {\cdot}7854 = 6{\cdot}25 \times {\cdot}7854 = 4{\cdot}9 =$ area of the valve.

$4{\cdot}9 \times 50 = 245$ lbs., whole weight on the valve.

$\frac{245}{8} = 30{\cdot}625$ lbs. = weight which must be put on at the end of the lever to give 50 lbs. per square inch.

$4{\cdot}9 \times 40 = 196 =$ whole weight on the valve.

$$\frac{196}{30{\cdot}625} = 6{\cdot}4 = \text{leverage.}$$

$6{\cdot}4 \times 3 = 19{\cdot}2 =$ the distance from the fulcrum the weight must be placed to have 40 lbs.

$24 - 19{\cdot}2 = 4{\cdot}8$; that is, the weight must be shifted in towards the fulcrum 4·8 inches to have 40 lbs. per inch; and for 30 lbs. per square inch, move it in 4·8 inches more, &c.

We must now take into account the weight of the lever, for when it is large and the valve is small a very sensible effect is produced on each square inch.

To find what Weight must be put on at the end of a Lever, to give any number of pounds pressure per square inch upon the Valve, the weight of the Lever being taken into consideration.

Rule.—Find the area of the valve by multiplying the square of the diameter by ·7854; then multiply this area by the number of pounds per square inch which you want upon the valve, and this product will give the whole weight upon the valve.

Next divide the whole length of the lever by the distance between the fulcrum and valve*, and the quotient will give the leverage which any weight will have when put on at the end of the lever.

Multiply this leverage by half the weight of the lever, and the product will give the pressure on the whole valve from the action of the lever alone; add to this product the weight of the valve, &c., and subtract the sum from the whole weight on the valve above mentioned; the remainder will give the weight which will be pressing on the valve from the action of the weight alone; and this, divided by the leverage, gives the weight itself.

* What is here meant by the distance between the fulcrum and valve is that part of the lever between the fulcrum and the point where the lever acts upon the valve.

The result would be the same if, instead of considering half the weight of the lever to act at the end, we should conceive the whole weight to act at the centre of gravity, the lever being supposed uniform.

Example 1.—Given the length of the lever, 24 inches; the distance between the fulcrum and valve, 3 inches; the weight of the valve, 3 lbs.; the weight of the lever, 3 lbs.: it is required to determine what weight must be put upon the end of the lever that it may press 30 lbs. per square inch, the diameter of the valve being 3 inches.

Now, $3^2 \times \cdot 7854 = 9 \times \cdot 7854 = 7$ square inches nearly, area of valve.

$7 \times 30 = 210$ lbs., whole weight on valve.

$\frac{24}{3} = 8$, leverage of the end of the lever.

$8 \times \frac{3}{2} = 12$ lbs., weight upon valve from the action of the lever alone.

$12 + 3 = 15$ lbs., weight of both lever and valve.

$210 - 15 = 195$ lbs., weight upon the valve from the action of the weight alone.

$\frac{195}{8} = 24 \cdot 375$ lbs., weight required to give a pressure of 30 lbs. per square inch on the valve.

To find now those points where the above-found weight will give 20 lbs. per inch, and also 10 lbs. per inch, we have only to invert the above operation; for we have the weight given, the valve, &c., and we have only to find the leverage; and the leverage multiplied by the distance between the fulcrum and valve, will give us the distance from the fulcrum the given weight must be put on.

Thus, $7 \times 20 = 140$ lbs., whole weight on valve.

$140 - 15 = 125$ lbs., weight upon the valve to be obtained from the action of the weight alone.

$$\frac{125}{24 \cdot 375} = 5 \cdot 128, \text{ leverage.}$$

$5 \cdot 128 \times 3 = 15 \cdot 384$ inches from the fulcrum for 20 lbs. pressure per inch.

Now, for 10 lbs. pressure,

$$7 \times 10 = 70 \text{ lbs., whole weight on valve.}$$

$70 - 15 = 55$ lbs., weight upon the valve from the action of the weight alone.

$$\frac{55}{24{\cdot}375} = 2{\cdot}256 \text{ leverage.}$$

$$2{\cdot}256 \times 3 = 6{\cdot}768 \text{ inches from the fulcrum.}$$

Example 2.—Given the length of the lever 16 inches, and its weight 2 lbs.; the distance between the fulcrum and valve, 2 inches; and the weight of the valve and spindle, $1\frac{1}{2}$ lb.: to find what weight must be put on at the end of the lever to press 40 lbs. per square inch upon the valve, the diameter of which is 2 inches.

The square of 2 is 4; hence,

$$\begin{array}{rl} {\cdot}7854 & \\ 4 & \\ \hline 3{\cdot}1416 & \text{square inches, area of the valve.} \\ 40 & \text{lbs.} \\ \hline 125{\cdot}6640 & \text{lbs.} = \text{weight on the whole valve.} \end{array}$$

$\frac{16}{2} = 8$, leverage which the weight will have at the end.

Now to consider half the weight of the lever to act at the end is the same as to consider the whole weight of the lever to act at its centre of gravity, the lever being uniform.

$$\begin{array}{ll} 1 \text{ lb.} & = \text{half the weight of the lever.} \\ 8 & = \text{leverage at the end of the lever.} \\ \hline 8 \text{ lbs.} & = \text{weight on the whole valve from the action of the lever.} \\ 1{\cdot}5 \text{ lbs.} & = \text{weight of valve, \&c.} \\ \hline 9{\cdot}5 \text{ lbs.} & = \text{weight on the valve from the action of both lever and valve.} \end{array}$$

$$125{\cdot}664 - 9{\cdot}5 = 116{\cdot}164 \text{ lbs.}$$

That is, the weight put on at the end of the lever must be such as to press 116·164 lbs. on the whole valve; but

the leverage of the weight is 8, therefore $\frac{1}{8}$th part of this weight will do. Thus, $116{\cdot}164 \div 8 = 14{\cdot}52$ lbs. = weight which must be put on at the end to press 40 lbs. per inch upon the valve.

Now to mark the lever where we will have 35 lbs., 30 lbs., 25 lbs., 20 lbs., &c., per square inch, we must proceed thus:—

3·1416 square inches, area of valve.

$3{\cdot}1416 \times 35 = 109{\cdot}956$ lbs., whole weight on valve.

$109{\cdot}956 - 9{\cdot}5 = 100{\cdot}456$, weight upon the valve from the action of the weight alone.

$\frac{100{\cdot}456}{14{\cdot}52} = 6{\cdot}918$, leverage which the weight must have.

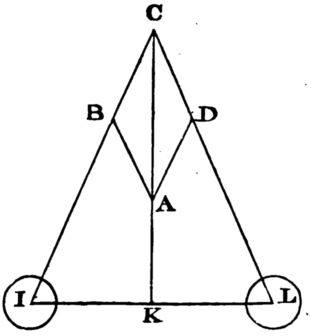
$6{\cdot}918 \times 2 = 13{\cdot}836$ = distance along the lever from the fulcrum.

$16 - 13{\cdot}836 = 2{\cdot}164$ inches = the distance which the weight must be moved in towards the valve.

And if you want 30 lbs. per square inch, move it in 2·164 inches further; and so on, as far as you please, making the division always 2·164 inches.

Rules and Examples on the Governor.

In calculations for the governor there are two conditions to be fulfilled. The first one is fulfilled by finding the position of the point A of the slider when the governor has the required angular velocity, or performs the number of revolutions required. To fulfil the other condition, the centrifugal force of the balls should be capable of regulating the supply of steam through the throttle-valve, when the velocity becomes greater than that with which the engine moves when doing its usual work, so as to bring the engine back to its proper speed.



Given the number of revolutions of the balls per minute, to find the distance of the point of suspension from the plane in which they revolve.

Rule.—Divide 35199 by the square of the number of revolutions per minute, and the quotient is the distance of the point of suspension of the balls from the plane in which they revolve.

Example.—What is the distance between the point of suspension of the balls of a governor and the plane in which they revolve, when it makes 25 revolutions per minute?

By the rule, $\frac{35199}{25^2} = \frac{35199}{625} = 56{\cdot}16$ inches.

To find the Range of the Balls or Radius of the Circle described by them.

Rule.—From the square of the length of the arm, subtract the square of the distance between the point of suspension and the plane of revolution, and the square root of the remainder will be the radius required.

Example.—If the length of the arms of a governor be 30 inches from the point of suspension to the centre of the ball, and the governor revolves 40 times in a minute, what will be the range of this governor?

Then, $\frac{35199}{40^2} = \frac{35199}{1600} = 22$ inches nearly,

the distance between the point of suspension and the plane of revolution.

Then $30^2 - 22^2 = 900 - 484 = 416$,

and $\sqrt{416} = 20{\cdot}4$ inches, the range required.

On the Crank.

The crank is used for converting a rectilinear into a rotary motion. In the crank, as applied in the steam engine, the effect which is produced is to the effect, were the force to act perpendicularly on the crank all the way round, as twice the diameter of the circle is to the circumference; in consequence of which, many practical men

have considered that there is a corresponding loss of power by using a crank; without ever considering that the piston or moving power only moves through twice the diameter of the crank's orbit, while the crank moves through its whole circumference. For, here the same principle holds good as in all other mechanical contrivances, viz., the power multiplied by the space which it passes over is equal to the weight or resistance multiplied by the space which it passes over.

This will be fully discussed afterwards on dynamical principles.

On Railways and Locomotive Engines.

In proceeding to give calculations for the speed, &c., of locomotive engines, we shall explain the motion of bodies on incline planes, or gradients, as they are generally called.

The theory of the incline plane is given in all its various bearings in *Hann's Theoretical and Practical Mechanics*, and the following is some of its most useful applications.

If a body be placed on an incline plane, its gravity is found by multiplying its weight by the fraction that represents the inclination of the plane; thus, if a body whose weight is 60 tons be placed upon a plane which rises 1 in 100, the gravity is $60 \times \frac{1}{100} = \frac{60 \times 2240}{100} = 1344$ lbs. If we suppose a train of 60 tons to be drawn up an incline plane of the above inclination, friction being 8 lbs. per ton, then $60 \times 8 = 480$ lbs. = the friction of the carriages.

And $\frac{60 \times 2240}{100} = 1344$ lbs. = the gravity of 60 tons on a plane inclined 1 in 100.

The sum is 1824 = the whole resistance from friction and gravity.

If a train descends an incline, then the difference between the friction and gravity gives the resistance.

In most planes ascended by locomotives, the inclination is so small that the pressure on the plane is very nearly the same as the weight of the body, and may be taken as that weight.

Let us suppose a train of 100 tons to ascend an incline plane that rises 1 in 80 at the rate of 20 miles per hour, what must be the horse power of the engine, friction being 8 lbs. per ton?

Since there are 5280 feet in one mile, then $5280 \times 20 = 105600$ feet per hour, and $\frac{105600}{60} = 1760$ feet per minute. The weight of the train in pounds is 224000. Now, as the rise of the rail is 1 in 80, we have the rise for 1760, $\frac{1}{80} \times 1760 = 22$ feet; hence the weight of the train is raised through 22 feet per minute in opposition to gravity, and the work done by gravity in each minute is the number of pounds multiplied by the number of feet it is raised through, or $224000 \times 22 = 4928000$.

The work absorbed by friction per minute is $8 \times 100 \times 1760 = 1408000$; therefore the whole work per minute is $4928000 + 1408000 = 6336000$, and $\frac{6336000}{33000} = 191{\cdot}8$ = the horse power.

Let us now suppose a train of 100 tons to descend an incline plane (or gradient) which has a rise of 1 in 500 at the rate of 70 miles per hour, to find the horse power, the friction being 8 lbs. per ton.

Now, 70 miles per hour is 6160 feet per minute, and $100 \times 8 \times 6160 = 4928000$ = friction per minute.

The weight of the train in pounds is $2240 \times 100 = 224000$; the rise being 1 in 500, the rise in 6160 is $6160 \times \frac{1}{500} = 12{\cdot}3$. The work due to gravity $= 224000 \times 12{\cdot}3 = 2755200$; but in this case the engine and gravity act in the same direction, viz. down the plane. The work done by gravity subtracted from the work done by the friction, is the work which the engine is required to do; hence

$4928000 - 2755200 = 2172800 =$ the work done by the engine per minute, hence the horse power $= \frac{2172800}{33000} = 66$.

On the Resistances to Locomotive Engines.

The locomotive engine has to overcome two different kinds of resistance, viz. the resistance of the air, and the escape of the steam, both of which vary with the velocity; hence it follows that the coefficient will not be the same for all velocities.

These resistances may be stated as follows: the resistance of the atmosphere, gravity, the friction of the axles, as well as that of the rims of the wheels upon the rails; also the friction of the moving parts of the engine, the resistance due to the passage of the steam through the steam ways and the blast pipe: the resistance due to the blast pipe is very great at great velocities. The atmospheric pressure, the resistance of which may be in round numbers estimated at 15 lbs. per square inch; the resistance of the air is difficult to determine: it is considered to vary as the square of the velocity; we shall, however, give the result of Pambour's experiments on this subject, and also examples to illustrate the theory.

The power of the locomotive engine is not to be estimated by the pressure of the steam in the boiler, and the diameter and length of stroke of the piston. In passing between the boiler and cylinder the elastic force of the steam is diminished before it reaches the cylinder, by the smallness of the apertures of the steam pipes through which it has to pass. This diminution is likewise more frequently produced by the evaporating power of the engine not being capable of keeping up a supply of steam to the cylinders of the elasticity equal to that in the boiler; therefore, the pressure upon the piston is less than that against the safety valve of the boiler, and this diminution of the elasticity of the steam in the cylinders, compared with that in the boiler, will, in many cases, be in the ratio

of the increase of velocity of the engine. Thus, supposing an engine capable of evaporating a certain quantity of water per hour, or converting it into a certain bulk or quantity of steam, of the elasticity indicated by the valve on the boiler, if this production of steam is sufficient to supply as many cylinders, full of steam of the density of that in the boiler, as shall be equal to the number of strokes per minute of the piston required to produce the given velocity, the elasticity of steam in the cylinder will be the same as that in the boiler, except that which is required to force the steam through the steam passages with the requisite velocity; and, consequently, the pressure on the piston will be nearly equal to that in the boiler. But if the velocity of the engine is such that the number of cylinders full of steam required is greater than the evaporation of the boiler can supply at the elasticity marked by the safety valve, then the elasticity in the cylinders is correspondingly diminished.

Example.

A railway train, weighing 90 tons, moves at the rate of 30 miles per hour upon a level, the resistance from friction 6 lbs. per ton, the resistance of the atmosphere 30 lbs. on the train when the rate is 10 miles per hour, the diameter of the driving wheel 6 feet, the area of the piston 100 square inches, the length of the stroke 18 inches, the resistance due to the blast pipe is 1·75 lbs. per square inch of the piston when the rate is 10 miles per hour: find the pressure of the steam, the evaporation of the boiler, the number of bushels of coals for a journey of 150 miles, supposing that one bushel of coals can evaporate 11 cubic feet of water.

$6 \times 90 = 540 =$ resistance from the friction.

$\left(\frac{30}{10}\right)^2 \times 30 = 270 =$ resistance of atmosphere against the train.

Whole resistance to the motion of the carriage

$$= 540 + 270 = 810 \text{ lbs.}$$

Space moved over by the driving wheel in one revolution

$$= 6 \times 3{\cdot}1416 = 18{\cdot}8496.$$

Work of the resistances in one revolution

$$= 18{\cdot}8496 \times 810 = 15268{\cdot}176..$$

Now, since each engine has two cylinders, and each piston makes two strokes during one revolution of the driving wheel, we must multiply the work done in one stroke by 4.

Hence, the work of one pound pressure per square inch on the pistons in one revolution of the driving wheel

$$= 1 \times 100 \times \frac{3}{2} \times 4 = 600.$$

The effective pressure upon one square inch of the piston, multiplied by the work of one pound per square inch pressure in one revolution, must be equal to the resistances in one revolution; hence we have the pressure on one square inch $= \frac{15268{\cdot}176}{600} = 25{\cdot}44$ lbs.

Pambour has found by experiment that the resistance of the blast pipe increases with the velocity; and the coefficient he gives is ·175, and at any given velocity is ·175 multiplied by that velocity; hence, for a velocity of 10 miles an hour, it is 1·75 lbs., for a velocity of 20 miles an hour it is 3·5 lbs., and for 30 miles per hour is 5·25 lbs.

$$\therefore 25{\cdot}44 + \tfrac{1}{7} \times 25{\cdot}44 + 5{\cdot}25 + 1 + 15 = 50{\cdot}3 \text{ lbs.}$$

Number of revolutions of the driving wheel per minute is

$$\frac{30 \times 5280}{6 \times 3{\cdot}1416 \times 60} = \frac{158400}{1130{\cdot}976} = 140.$$

The number of strokes of the piston per minute is

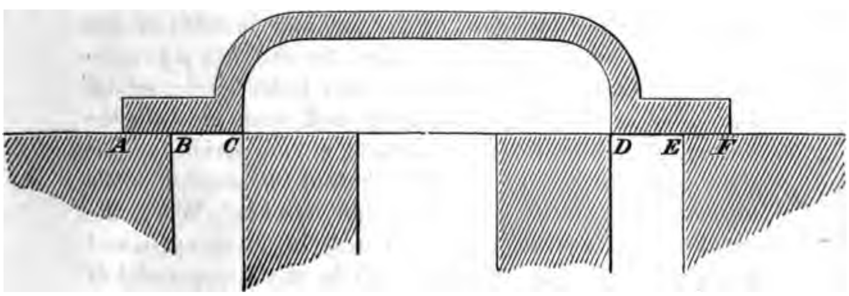
$$4 \times 140 = 560.$$

The volume of steam discharged per minute $= \frac{100}{144} \times \frac{3}{2} \times 560 = 583$ cubic feet, but by the table 1 foot of water produces 552 feet of steam at 50 lbs. pressure; hence the number of cubic feet evaporated per minute is $\frac{583}{552} = 1{\cdot}05$, and as 1 bushel of coals can evaporate 11 cubic feet of

water, the number of bushels of coals used in one minute $= \frac{1 \cdot 05}{11}$, and for 5 hours, which it requires to go over 150 miles, we have number of bushels of coals for 5 hours is $\frac{1 \cdot 05}{11} \times 60 \times 5 = 29$ nearly.

On the Slide Valve, taking into account the Lap and Lead.

When the slide has neither lap nor lead, the breadth of the slide face is equal to that of the steam port, and the travel of the slide is evidently equal to twice the breadth of the steam port; but when the slide has lap, then the length of the stroke or travel of the slide is equal to double the lap, together with double the steam port. In the annexed figure, which represents the slide at half stroke,



AB is the lap, and BC or DE equals the breadth of the steam port; when the slide begins to move from one extremity of its stroke, the point F is then at D; when it has moved half its travel, or comes into the position represented in the figure, it is clear that the point F has moved over the space DF, which is equal to the steam port added to the lap: now, the slide moving through the other half of its travel, will bring the point A to C, which distance is also equal to

the steam port added to the lap; hence the whole travel of the slide is equal to twice the breadth of the steam port, added to twice the lap.

The following method of showing the positions of the piston, corresponding to the various openings of the steam and exhausting ports, has been transmitted to us by the eminent engineer, Mr. Amos, of the well-known firm of Easton and Amos. It is given in as simple a manner as the subject seems to admit of, for any working man may, with rule and compasses, draw the figure to any convenient scale; and, as regards locomotive engines, he might draw it to the full size, or nearly so.

Geometrical Construction.

From a scale of equal parts, take the length of the stroke of piston, divided into feet and inches, and with radius equal to half the stroke, describe the circle A C D, which represents the crank orbit. Also from a scale of equal parts, the scale being as large as practicable, with the radius O P, equal to the lap of the slide, describe the circle *o* P Q, and with radius O*p*, equal to the lap and breadth of one steam port, describe the circle *p q a*—the distance *p q* represents the travel of the slide; on this latter circle, set off *l m* equal to the lead of the slide, and from *m* draw the line *m n*, touching the circle *o*P Q at P, and parallel to this drawn the line A A′, and A will represent the position of the crank-pin when the slide is at half stroke. When the crank-pin arrives at B the slide will begin to open, and when it arrives at C the piston will be at the upper end of its stroke, and the steam port will be open a distance equal to *l m*, and taking the steam on the upper side of the piston to commence the down stroke. To ascertain the position of the crank and piston when the steam port is full open, from *m* and *n* as centres, describe the arcs *d e* and *f g*, their point of intersection *r*, in the circle A C D, will be the position of the crank-pin, and the line *r s*, parallel to C H, will be the position of the piston at that time. When the crank-pin arrives at the point *t*, the slide will have returned

to the line $m\,n$ and the steam will be cut off, and the line t T, drawn parallel to C H, will be the position of the piston, showing the portion of the stroke performed. When the crank-pin arrives at b the steam port begins to open for admission of steam to the under side of the piston to commence the up-stroke of the piston when the crank arrives at D.

To ascertain the position of the slide with respect to the piston at any portion of its stroke, as, for example, when the piston has performed six inches of its stroke, draw I i parallel to H C, draw the line $i\,e'$, which if produced would pass through the centre O, and from the point e' draw $e'h$ at right angles to $m\,n$, then the distance $e'\,h$ will be equal to the breadth the steam port is open at that time. Again, if the piston has travelled $33\frac{1}{2}$ inches, draw $v\,w$ parallel to H C, and from w draw $w\,x$ radiating to the centre O, and parallel to $m\,n$ draw xy, then the upper port will be closed, and the slide will have travelled the distance pk; see also Fig. 4, Plate 3.

In the foregoing the slide has been considered with respect to the admission of the steam; we shall consider it with respect to the exhaustion of the steam from the opposite side of the piston.

Draw the length of stroke and crank orbit as before, also draw the line A A′, making the same angle with C D as before, with the lap on the exhausting side as radius, describe the circle P o Q, and parallel with the line A A′ draw the lines xy and $x'y'$, then the distance Pp' will represent the breadth of the steam ports a to the cylinder (figures 3, 4, 5, 6), and p P equal the breadth of the exhaust port b. When the crank is at A, the slide is at the middle of its stroke; when it arrives at B the steam begins to exhaust from the under-side of the piston; and when the crank arrives at C, the piston being at this time at the upper end of its stroke, the slide will be open the distance $l\,m$; and when the crank arrives at c, the same port will be open the distance $y'm'$, equal the width p'P of the steam ports; when the crank arrives at r, the slide will be at the

end of its travel, shown in Fig. 4, *p* P being equal to the exhaust ports *b* (figures 3, 4, 5, 6), and *s* will be the position of the piston in its down-stroke. It will be seen that during the time the piston is travelling from *c* to *c′*, the bottom port *p′* P has been fully open, to allow the steam to escape from the under-side of the piston. When the crank arrives at *w*, the lower port is shut, and the slide is in the position shown in Fig. 6.

Rules and Examples for the Lap and Lead of the Slide.

If the lead, lap, lengths of the stroke of the slide and piston be given, to find where the steam is cut off:—

Rule.—Add the lap and half the lead of the slide together, divide the sum by half the stroke of the slide, multiply the square of this quotient by the length of the stroke of the piston, subtract this product from the length of the stroke, and the remainder will be the distance travelled by the piston before the steam is cut off. Taking the dimensions the same as in the last problem, observing that they are all taken in inches,

$$40 \times \left(\frac{1{\cdot}8557 + {\cdot}1}{3}\right)^2 = 40 \times \left(\frac{1{\cdot}9557}{3}\right)^2 = ({\cdot}6519)^2 \times 40$$

$= 17$ inches.

And $40 - 17 = 23$ inches the piston has moved over before the steam was cut off.

Given the length of the stroke of the piston, the distance moved by the piston before the steam is cut off, the length of the stroke of the slide and the lead: to find the lap or cover, subtract the distance moved by the piston before the steam is cut off, from the whole stroke of the piston; divide the remainder by the length of the stroke of the piston, and take the square root of the quotient; multiply this root by half the length of the stroke of the

slide, from this product subtract half the lead, and the remainder will be the lap required.

Example.—The length of the stroke of an engine is 40 inches, the steam is cut off at 23 inches, the travel or stroke of the valve 6 inches, and the lead $\frac{1}{5}$ of an inch: find the lap.

Now, $40 - 23 = 17$ inches, the distance the piston has to travel after the steam is cut off.

then $\sqrt{\frac{17}{40}} = \sqrt{\cdot 425} = \cdot 6519$; this, multiplied by half the travel of the slide, $\cdot 6519 \times 3 = 1\cdot 9557$,

and $1\cdot 9557 - 1 = 1\cdot 8557$ inch, the lap required.

To find at what part of the Stroke the steam will be cut off, when the breadth of the Steam Port, the Lap, and Lead are given.

Divide the breadth of the steam port by half the travel of the slide, and the quotient is the versed sine of an arc; subtract the lead from the breadth of the port, and divide the remainder also by the travel of the slide, and the quotient is the versed sine of another arc; from the table of versed sines find the corresponding arcs. If the sum of these arcs be less than 90°, multiply the versed sine of their sum by half the stroke in inches, and the product will be the distance of the piston from the commencement of the stroke when the steam is cut off. If the sum of the two arcs exceed 90°, subtract that sum from 180°, and the versed sine of the difference multiplied by half the stroke is equal to the distance of the piston from the end of the stroke when the steam is cut off.

The stroke of a piston, 60 inches; the width of the steam port, 3 inches; lap on the steam side, $2\frac{1}{2}$ inches; lap on the exhaust side, $\frac{1}{8}$ inch; and lead $\frac{1}{2}$ inch: required the point of the stroke at which steam will be cut off.

Here $\frac{3}{3 + 2\cdot 5} = \cdot 5454 =$ versed sine of $62° 58'$ (arc the first),

and $\frac{3 - \cdot 5}{3 + 2\cdot 5} = \cdot 4545 =$ versed sine of 56° 57′ (arc the second).

Then 62° 58′ + 56° 57′ = 119° 55′; and 180° − 119° 55′ = 60° 5′ = arc of versed sine, ·5012. ·5012 × 30 = 15·036 inches, = distance of the piston from the end of the stroke when the steam is cut off.

To find at what part of the Stroke the exhaustion ceases.

Subtract the lap on the exhausting side from half the travel of the slide, and divide the remainder by half that travel, and the quotient is the versed sine of an arc; from the table find the corresponding arc, and add it to the second arc in the preceding rule; the versed sine of the difference between their sum and 180°, multiplied by half the stroke, gives the distance of the piston from the end of the stroke when the exhaustion ceases.

Example.—3 + 2·5 = 5·5 = half the slide's travel; and $\frac{5\cdot 5 - \cdot 125}{5\cdot 5} = \cdot 9772 =$ versed sine of arc 88° 42′ (arc the third); then 88° 42′ + 56° 57′ (arc the second) = 145° 39′; and 180° − 145° 39′ = 34° 21′ = arc of versed sine, ·1743. ·1743 × 30 = 5·229 inches = the distance of the piston from the end of its stroke when exhaustion ceases.

Find the distance of the Piston from the end of the Stroke when exhaustion commences.

Subtract the second arc from the third, and multiply the versed sine of their difference by half the stroke; the product will be the distance required.

Using the values in the two preceding examples,

88° 42′ − 56° 57′ = 31° 45, the versed sine of which is ·1496, and ·1496 × 30 = 4·488 inches, the required distance.

Find the distance of the Piston from the end of the Stroke when the steam is admitted for the Return Stroke.

Multiply the versed sine of the difference of the first and second arcs by half the length of the stroke, and the product will be the distance required.

The first arc is 62° 58′, and the second is 56° 57′.

62° 58′ − 56° 57′ = 6° 1′, the versed sine of which is ·0055; and ·0055 × 30 = ·165 inch.

Rule.—To find the proportions of the steam-lap and lead, the points of the stroke where steam is cut off, and re-admitted for the return stroke being known. When the steam is cut off before half-stroke, divide the portion of the stroke performed by the piston, by half the stroke, and call the quotient versed sine. Likewise, divide the distance of the piston from the end of its stroke when steam is re-admitted for the return stroke, by half the stroke, and call that quotient versed sine. Find their respective arcs, and also the versed sines of half their sum and half their difference. The width of the steam port in inches, divided by the versed sine of half their sum, equals half the travel of the slide, and half the travel, minus the width of port, equals the lap. The difference of the two versed sines last found, multiplied by half the travel of the slide, equals the lead. When the steam is to be cut off after half-stroke, divide the distance of the piston from the end of its stroke, by half the stroke; call the quotient versed sine, and subtract its corresponding arc from 180 degrees. Divide the distance the piston has to move when the steam is admitted for the return stroke, by half the stroke; call the quotient versed sine, and find its corresponding arc.

Then proceed with the two arcs thus found as in the former case.

Example 12.—The stroke of a piston is 60 inches, the width of steam-port 3 inches, distance of the piston from the end of its stroke, when steam is cut off, 15·036 inches, and when steam is admitted for the return stroke ·165 inch; required the lap and lead.

$$\text{Here } \frac{15\cdot036}{30} = \cdot5012 = \text{versed sine of arc } 60^\circ\ 5';$$

$$\text{and } 180^\circ - 60^\circ\ 5' = 119^\circ\ 55'.$$

$$\text{Then } \frac{\cdot165}{30} = \cdot0055 = \text{versed sine } 6^\circ\ 1'.$$

$$119^\circ 55' + 6^\circ\ 1' = 125^\circ\ 56';\ 119^\circ\ 55' - 6^\circ\ 1' = 113^\circ\ 54.'$$

$$\frac{125^\circ\ 56'}{2} = 62^\circ\ 58' = \text{arc of versed sine } \cdot5454;$$

$$\frac{113^\circ\ 54'}{2} = 56^\circ\ 57' = \text{arc of versed sine } \cdot4545.$$

$$\frac{3}{\cdot5454} = 5\cdot5 \text{ inches} = \text{half the slide's travel};$$

$$\text{and } 5\cdot5 - 3 = 2\cdot5 = \text{lap}.$$

$$\cdot5454 - \cdot4545 = \cdot0909;\ \text{and } \cdot0909 \times 5\cdot5 = \cdot5 \text{ inches} = \text{lead}.$$

A Table of Multipliers to find the Lap and Lead, when the Steam is to be cut off at ½ to ⅞ths of the Stroke.

The lap must be equal to the width of the steam-port multiplied by Col. 1.

The lead must be equal to the width of the steam-port multiplied by Col. 2.

Half-Stroke.		Five-Eighths of the Stroke.		Three-Fourths of the Stroke.		Seven-Eighths of the Stroke.		The distance of the piston from the end of its stroke when steam is readmitted for the return stroke being equal to half the stroke multiplied by
1	2	1	2	1	2	1	2	
Lap	Lead	Lap	Lead	Lap	Lead	Lap	Lead	
2·41	·000	1·58	·000	1·000	·000	·540	·000	·00000
2·16	·145	1·41	·124	·893	·105	·477	·089	·00208
2·06	·198	1·35	·170	·851	·146	·450	·123	·00416
1·94	·268	1·27	·231	·795	·200	·413	·170	·00833
1·84	·318	1·21	·276	·754	·240	·385	·204	·01250
1·77	·358	1·16	·312	·723	·271	·363	·232	·01666
1·71	·391	1·12	·342	·691	·299	·344	·257	·02083
1·65	·420	1·08	·368	·668	·322	·327	·277	·02500
1·60	·444	1·05	·391	·644	·343	·313	·296	·02916
1·56	·467	1·02	·412	·623	·362	·298	·313	·03333
1·48	·505	·968	·449	·586	·396	·273	·343	·04166
1·41	·540	·921	·480	·554	·425	·251	·370	·05000
1·35	·570	·881	·508	·526	·451	·232	·393	·05833
1·30	·595	·844	·532	·500	·473	·215	·414	·06666
1·25	·617	·810	·554	·476	·495	·198	·434	·07500
1·21	·638	·779	·572	·454	·514	·183	·452	·08333
1·17	·657	·751	·592	·434	·532	·160	·468	·09166
1·13	·674	·724	·607	·415	·548	·156	·483	·10000

Example of its application.—Stroke 36 inches, width of port 2 inches, steam to be cut off at half-stroke, distance of the piston from the end of its stroke when steam is re-admitted for the return stroke, 1·5 inch.

$\frac{1·5}{18} = 0833$. Find that number, or the one nearest to it, in the right-hand or last column, and take out the multipliers on the same line under the head Half-Stroke.

$$\text{Then } 2 \times 1·21 = 2·42 \text{ inches} = \text{the lap.}$$
$$\text{And } 2 \times ·638 = 1·276 \text{ inch} = \text{the lead.}$$

5 ft. PASSENGER.

TABLE showing the Working of the Slide Valves of the "Elephant" Engine, fitted with Expansion Gear, 15½ in. Cylinders, 18 in. Stroke, Dec. 1849.

Outside Lap of Slide, ¾ in. | Dimen. of Steam Ports, 1¼ in. } 11 in.
Inside „ „ 1/16 | Do. of Exhaust Ports, 2¼ }

No. of Notch.	Travel of Slide.	Lead of Slide.	Steam cut off.		Exhaust opens.		Compression begins.		Slide opens.		Remarks.
			Front Stroke.	Back Stroke.	Front Stroke.	Back Stroke.	Front Stroke.	Back Stroke.	Front Stroke.	Back Stroke.	
1	4	5/16	15¾	14⅜	17½	16¾	17¼	17½	1¼	1¼	Inside diame-
2	3¾	5/16	15⅛	14⅛	17⅜	17¾	16¼	17⅜	1⅛	1⅛	ter of Blast
3	3¼	¼	14⅛	12⅝	17	16⅛	16⅝	16	⅞	⅞	Pipe, 3¼
4	2¾	3/16	13⅜	12	16⅝	15⅞	16¼	15⅝	⅝	⅝	inches.
5	2¼	⅛	11½	10⅝	16⅜	15¼	15⅞	14⅞	⅜	⅜	
6	2¼	⅛	10	9½	15¾	14⅝	15¼	14⅜	...	...	
7	2	1/16	7⅞	7⅜	15	13¾	14¾	13	¼	¼	

N.B. The Front Stroke is the Piston moving from the Buffer Bar towards the Fire Box.

6 ft. Passenger.

Table showing the Working of the Slide Valves of the "Wolf" Engine, fitted with Common Gear, 15 in. Cylinders, 18 in. Stroke, 19th May, 1852.

Outside Lap of Slide, $\frac{3}{4}$ in.	Dimen. of Steam Ports,	$1\frac{1}{4}$ in.	11 in.
Inside " " $\frac{1}{16}$	Do. of Exhaust Ports,	$11\frac{5}{16}$	

No. of Notch.	Travel of Slide.	Lead of Slide.	Steam cut off.		Exhaust opens.		Compression begins.		Slide opens.		Remarks.
			Front Stroke.	Back Stroke.	Front Stroke.	Back Stroke.	Front Stroke.	Back Stroke.	Front Stroke.	Back Stroke.	
1	$3\frac{3}{4}$	$\frac{5}{16}$	$14\frac{7}{16}$	$13\frac{5}{8}$	$16\frac{3}{4}$	$16\frac{3}{8}$	$16\frac{1}{2}$	$16\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{5}{32}$	Inside diameter of Blast Pipe, $3\frac{7}{8}$ inches.
2	...	...	...	...	...	...	...	...	...	...	
3	...	...	...	...	...	...	...	...	...	...	
4	...	...	...	...	...	...	...	...	...	...	
5	...	...	...	...	...	...	...	...	...	...	
6	...	...	...	...	...	...	...	...	...	...	
7	...	...	...	...	...	...	...	...	...	...	

N.B. The Front Stroke is the Piston moving from the Buffer Bar towards the Fire Box.

7 ft. PASSENGER.

TABLE showing the Working of the Slide Valves of the "Milo" Engine, fitted with Expansion Gear, 15 in. Cylinders, 20 in. Stroke, March, 1852.

Outside Lap of Slide, $\frac{13}{16}$ in.	Dimen. of Steam Ports, $1\frac{1}{4}$ in.	$10\frac{7}{8}$ in.
Inside „ „ $\frac{1}{16}$	Do. of Exhaust Ports, $2\frac{3}{8}$	

No. of Notch.	Travel of Slide.	Lead of Slide.	Steam cut off.		Exhaust opens.		Compression begins.		Slide opens.		Remarks.
			Front Stroke.	Back Stroke.	Front Stroke.	Back Stroke.	Front Stroke.	Back Stroke.	Front Stroke.	Back Stroke.	
1	$4\frac{1}{4}$	bare $\frac{3}{8}$	$16\frac{3}{4}$	$15\frac{1}{16}$	$19\frac{1}{2}$	$17\frac{7}{8}$	$18\frac{7}{8}$	$17\frac{5}{8}$	$1\frac{1}{4}$	$1\frac{1}{4}$	Inside diameter of Blast Pipe, $3\frac{7}{8}$ inches.
2	$3\frac{7}{8}$	bare $\frac{3}{8}$	$16\frac{1}{8}$	$14\frac{3}{8}$	19	$17\frac{5}{8}$	$18\frac{3}{4}$	$17\frac{5}{16}$	$1\frac{1}{8}$	$1\frac{1}{8}$	
3	$3\frac{5}{8}$	$\frac{5}{16}$	$15\frac{1}{8}$	$13\frac{7}{8}$	$18\frac{3}{4}$	$17\frac{3}{8}$	$18\frac{1}{2}$	$16\frac{7}{8}$	1	full 1	
4	$3\frac{1}{8}$	$\frac{1}{4}$	$14\frac{3}{8}$	$12\frac{5}{8}$	$18\frac{5}{8}$	17	$18\frac{1}{4}$	$16\frac{3}{8}$	$\frac{3}{4}$	$\frac{3}{4}$	
5	$2\frac{7}{8}$	$\frac{3}{16}$	$13\frac{7}{8}$	$11\frac{5}{8}$	$18\frac{3}{8}$	$16\frac{1}{2}$	$17\frac{7}{8}$	$15\frac{7}{8}$	$\frac{5}{8}$	$\frac{5}{8}$	
6	$2\frac{5}{8}$	bare $\frac{3}{16}$	$11\frac{1}{2}$	$9\frac{7}{8}$	$17\frac{1}{4}$	$15\frac{3}{4}$	$17\frac{3}{4}$	$15\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	
7	$2\frac{3}{8}$	$\frac{1}{8}$	$8\frac{5}{8}$	$7\frac{3}{4}$	$17\frac{1}{8}$	$14\frac{1}{2}$	$16\frac{1}{2}$	14	$\frac{3}{8}$	$\frac{3}{8}$	

N.B. The Front Stroke is the Piston moving from the Buffer Bar towards the Fire Box.

7 ft. Passenger.

Table showing the Working of the Slide Valves of the "Peri" Engine, fitted with Expansion Gear, 16 in. Cylinders, 24 in. Stroke, April, 1852.

Outside Lap of Slide, $1\frac{1}{16}$ in.	Dimen. of Steam Ports,	$1\frac{7}{16}$ in.	13 in.
Inside " " $\frac{1}{8}$	Do. of Exhaust Ports,	$2\frac{3}{4}$	

No. of Notch.	Travel of Slide.	Lead of Slide.	Steam cut off.		Exhaust opens.		Compression begins.		Slide opens.		Remarks.
			Front Stroke.	Back Stroke.	Front Stroke.	Back Stroke.	Front Stroke.	Back Stroke.	Front Stroke.	Back Stroke.	
1	$4\frac{7}{8}$	$\frac{9}{16}$	$17\frac{3}{16}$	$16\frac{1}{8}$	$21\frac{3}{4}$	$21\frac{1}{4}$	$20\frac{15}{16}$	$20\frac{1}{8}$	$1\frac{5}{16}$	$1\frac{3}{8}$	Inside diame-
2	$4\frac{3}{8}$	$\frac{7}{16}$	$16\frac{1}{4}$	$15\frac{3}{8}$	$21\frac{1}{2}$	21	$20\frac{9}{16}$	$19\frac{7}{8}$	$1\frac{1}{16}$	$1\frac{1}{8}$	ter of Blast
3	$3\frac{15}{16}$	$\frac{3}{8}$	$14\frac{13}{16}$	14	21	$20\frac{5}{8}$	$19\frac{7}{8}$	$19\frac{5}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	Pipe, $4\frac{3}{8}$
4	$3\frac{1}{2}$	$\frac{5}{16}$	$12\frac{5}{8}$	$12\frac{3}{16}$	$20\frac{5}{16}$	$19\frac{7}{8}$	19	$18\frac{5}{16}$	$1\frac{1}{16}$	$\frac{3}{4}$	inches.
5	$3\frac{1}{4}$	$\frac{5}{16}$	$10\frac{1}{2}$	$10\frac{3}{16}$	$19\frac{1}{2}$	$19\frac{1}{8}$	$17\frac{3}{4}$	$17\frac{1}{4}$	$\frac{1}{2}$	$\frac{9}{16}$	
6	3	...	$8\frac{1}{8}$	$7\frac{5}{8}$	$18\frac{3}{8}$	$17\frac{3}{8}$	$16\frac{3}{8}$	$15\frac{11}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	
7	$2\frac{5}{8}$	$\frac{1}{4}$	$4\frac{9}{16}$	$4\frac{3}{16}$	$15\frac{7}{8}$	$15\frac{1}{4}$	$13\frac{1}{2}$	$11\frac{5}{8}$	$\frac{1}{4}$	$\frac{5}{16}$	

N.B. The Front Stroke is the Piston moving from the Buffer Bar towards the Fire Box.

8 ft. Passenger.

Table showing the Working of the Slide Valves of the "Rover" Engine, fitted with Expansion Gear, 18 in. Cylinders, 24 in. Stroke, September, 1850.

Outside Lap of Slide, 1 in.	Dimen. of Steam Ports, 2 in.	13 in.
Inside „ „ $\frac{1}{32}$	Do. of Exhaust Ports, $3\frac{1}{2}$	

No. of Notch.	Travel of Slide.	Lead of Slide.	Steam cut off.		Exhaust opens.		Compression begins.		Slide opens.		Remarks.
			Front Stroke.	Back Stroke.	Front Stroke.	Back Stroke.	Front Stroke.	Back Stroke.	Front Stroke.	Back Stroke.	
1	$4\frac{3}{4}$	$\frac{3}{8}$	$18\frac{1}{4}$	$17\frac{1}{2}$	$22\frac{1}{16}$	$21\frac{7}{16}$	22	$21\frac{3}{8}$	$1\frac{3}{16}$	$1\frac{3}{8}$	Inside diame-
2	$4\frac{3}{8}$	...	$16\frac{7}{8}$	$16\frac{5}{8}$	$21\frac{1}{2}$	$20\frac{3}{4}$	$21\frac{1}{4}$	$20\frac{7}{8}$	$1\frac{3}{16}$	$1\frac{1}{4}$	ter of Blast
3	$3\frac{11}{16}$	...	$14\frac{7}{8}$	$14\frac{3}{16}$	$20\frac{11}{16}$	20	$20\frac{7}{16}$	$19\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	Pipe, $5\frac{1}{4}$
4	$3\frac{3}{8}$	...	$11\frac{7}{8}$	12	$19\frac{1}{2}$	$18\frac{9}{16}$	$19\frac{3}{8}$	$18\frac{5}{16}$	$\frac{5}{8}$	$\frac{3}{4}$	inches.
5	$3\frac{1}{16}$	...	$8\frac{5}{8}$	9	$17\frac{3}{4}$	$17\frac{1}{8}$	$17\frac{3}{8}$	$16\frac{7}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	
6	$2\frac{7}{8}$	...	$5\frac{1}{2}$	$5\frac{7}{8}$	$15\frac{5}{16}$	15	$14\frac{7}{8}$	$14\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{8}$	
7	...	...	...	...	...	...	...	...	...	...	

N.B. The Front Stroke is the Piston moving from the Buffer Bar towards the Fire Box.

5 ft. COUPLED GOODS ENGINE.

TABLE showing the Working of the Slide Valves of the "Jason" Engine, fitted with Expansion Gear, 16 in. Cylinders, 24 in. Stroke, Feb. 1852.

Outside Lap of Slide, $\frac{3}{4}$ in.	Dimen. of Steam Ports, $1\frac{1}{2}$ in. } 13 in.
Inside „ „ $\frac{1}{16}$	Do. of Exhaust Ports, $2\frac{3}{4}$ } 13 in.

No. of Notch.	Travel of Slide.	Lead of Slide.	Steam cut off. Front Stroke.	Steam cut off. Back Stroke.	Exhaust opens. Front Stroke.	Exhaust opens. Back Stroke.	Compression begins. Front Stroke.	Compression begins. Back Stroke.	Slide opens. Front Stroke.	Slide opens. Back Stroke.	Remarks.
1	$4\frac{1}{4}$	$\frac{3}{8}$	20	19	$22\frac{1}{2}$	22	$22\frac{3}{4}$	$22\frac{3}{4}$	$1\frac{3}{4}$	$1\frac{5}{8}$	Inside diameter of Blast Pipe, 4 inches.
2	$3\frac{1}{2}$	$\frac{5}{16}$	19	$17\frac{1}{2}$	$22\frac{1}{8}$	$21\frac{1}{4}$	$22\frac{3}{8}$	$21\frac{1}{2}$	1	1	
3	$2\frac{3}{4}$	$\frac{1}{4}$	$17\frac{3}{4}$	$16\frac{1}{4}$	$21\frac{5}{8}$	$20\frac{3}{4}$	$22\frac{1}{8}$	$21\frac{1}{4}$	$\frac{7}{8}$	$1\frac{3}{16}$	
4	$2\frac{5}{8}$	$\frac{3}{16}$	$16\frac{1}{2}$	$15\frac{1}{4}$	$21\frac{1}{4}$	$20\frac{3}{8}$	$21\frac{3}{4}$	21	$\frac{5}{8}$	$\frac{5}{8}$	
5	$2\frac{1}{2}$	$\frac{1}{8}$	15	$13\frac{1}{2}$	$20\frac{1}{2}$	$19\frac{1}{2}$	$21\frac{1}{2}$	$18\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	
6	$2\frac{1}{8}$	$\frac{1}{8}$	14	11	20	$18\frac{1}{2}$	$20\frac{3}{4}$	$19\frac{1}{2}$	$\frac{3}{8}$	$\frac{3}{8}$	
7	2	$\frac{5}{8}$	11	$9\frac{1}{8}$	$18\frac{1}{2}$	$17\frac{1}{2}$	$17\frac{1}{8}$	$18\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	

N.B. The Front Stroke is the Piston moving from the Buffer Bar towards the Fire Box.

5 ft. Coupled Goods Engine.

Table showing the Working of the Slide Valves of the "Giaour" Engine, fitted with Expansion Gear, 17 in. Cylinders, 24 in. Stroke, May, 1852.

Outside Lap of Slide, 1 in.	Dimen. of Steam Ports, $1\frac{3}{4}$ in. } 13 in.
Inside " " $\frac{1}{16}$	Do. of Exhaust Ports, $3\frac{1}{4}$ }

No. of Notch.	Travel of Slide.	Lead of Slide.	Steam cut off.		Exhaust opens.		Compression begins.		Slide opens.		Remarks.
			Front Stroke.	Back Stroke.	Front Stroke.	Back Stroke.	Front Stroke.	Back Stroke.	Front Stroke.	Back Stroke.	
1	$4\frac{1}{2}$	$\frac{5}{16}$	$18\frac{7}{8}$	$19\frac{1}{2}$	$22\frac{1}{4}$	22	22	$21\frac{3}{4}$	$1\frac{1}{16}$	$1\frac{1}{2}$	Inside diameter of Blast Pipe, $5\frac{1}{2}$ inches.
2	$4\frac{3}{8}$	...	$17\frac{1}{2}$	$17\frac{7}{16}$	$21\frac{3}{4}$	$21\frac{5}{8}$	$21\frac{1}{2}$	$21\frac{5}{16}$	1	$1\frac{7}{16}$	
3	$3\frac{15}{16}$	...	$16\frac{1}{4}$	$16\frac{3}{8}$	$21\frac{5}{16}$	21	$21\frac{1}{16}$	$20\frac{9}{16}$	$1\frac{3}{16}$	$1\frac{3}{16}$	
4	$3\frac{5}{8}$	...	$14\frac{3}{8}$	15	$20\frac{7}{8}$	$20\frac{1}{2}$	$20\frac{3}{16}$	$20\frac{3}{8}$	$\frac{11}{16}$	1	
5	$3\frac{5}{16}$	...	12	$13\frac{1}{4}$	$19\frac{7}{8}$	$19\frac{5}{16}$	$19\frac{1}{2}$	$19\frac{5}{16}$	$\frac{9}{16}$	$\frac{13}{16}$	
6	$3\frac{1}{16}$	...	$9\frac{1}{8}$	$11\frac{3}{16}$	$18\frac{3}{4}$	$18\frac{9}{16}$	$18\frac{1}{4}$	$18\frac{1}{4}$	$\frac{7}{16}$	$\frac{5}{8}$	
7	...	...	...	...	...	...	...	...	...	...	

N.B. The Front Stroke is the Piston moving from the Buffer Bar towards the Fire Box.

The physical condition of bodies is determined by the action of two forces acting in opposite directions; viz. the pressure exerted upon them, and heat communicated to them. We may admit, as a general principle, that all bodies exist in three different states—solid, liquid, and gaseous; under the influence of the varied action of the two forces above-mentioned, increase of pressure tending to induce the solid state, and that of temperature inducing the gaseous state.

Water, which is liquid under the ordinary pressure and temperature of the atmosphere, assumes the gaseous state either by diminishing the pressure or by increasing the temperature in a certain ratio. Under the atmospheric pressure of 14·8 lbs. per square inch, and at a temperature of 212° Fahr., it becomes gaseous, or evaporates; under a higher pressure it will evaporate only at a higher temperature, and with a less pressure the evaporation will take place at a less temperature. According to the law by which these phenomena are governed, the increments of pressure being supposed regular, the respective increments of temperature at which evaporation takes place will not be regular.

If, for instance, the boiling points be denoted by T and T′ at 30 and 35 lbs. per square inch, and if at 25 and 30 lbs. by T″ and T, then, although the differences $30 - 25$ and $35 - 30$ be equal, yet the differences $T - T''$ and $T' - T$ will not be equal.

The law of the increments of temperature corresponding to equal increments of pressure cannot be expressed by a simple formula; the following Table will be found very useful.

In this Table the first and second columns contain the pressures expressed in atmospheres and in pounds per square inch, the third indicates the corresponding temperature of the steam; the other columns show other properties of steam calculated partly from the preceding columns and partly from the density of the steam at a certain pressure ascertained by experiment.

A quantity of steam being inclosed in a vessel where water is also present, the temperature will bear some relation to the pressure, and the temperature, although it may be above what is indicated by that relation, cannot fall below it. Now let a part of that steam be put into a separate vessel, and let the vessel be increased in volume, there being no escape of steam or temperature from it, then the pressure will become less, and as the specific heat of gases increases as the pressure diminishes, there must

also be a fall of temperature. It is upon this principle of the difference of specific heat of gaseous bodies at different pressures that their temperature is raised by reducing their bulk.

Let us take a certain volume of steam at 212° and at one atmosphere, and compress it till it has a pressure of three atmospheres; supposing that there is no loss of heat in the experiment, the elevation of temperature which follows will be greater than, equal to, or less than the difference of 212° and 275°, the latter being the temperature given in the Table opposite to three atmospheres.

If it be equal, the fall of temperature which results from the increase of volume which we have alluded to above, will be such, that when the pressure diminished by the expansion is one atmosphere, the temperature of the steam is exactly 212°. If it be less, the temperature of the steam after expansion will be between 275° and 212°. If it be greater, the final temperature will be less than 212°, if the steam could remain in the gaseous form; but since this cannot take place, except by raising the temperature to balance the pressure tending to reduce the steam to water, it follows, in the last case, that when a fall of temperature takes place without a corresponding diminution of pressure, condensation must ensue.

The latent heat evolved from the condensed vapour will be immediately absorbed by that portion which remains as steam; the process of condensation will cease immediately, and will be resumed only when a repeated increment of volume causes a further decrement of temperature.

Supposing this process to be repeated, it is clear that when the steam comes to 212° there will be produced a certain quantity of water.

In order to produce these phenomena, as we have before observed, it is necessary that the heat evolved by the steam passing from 275° to 212° be insufficient to supply the additional absorption arising from the difference of the specific heats. Now that this is the case is evident from the known laws of the specific heat of gases.

Although we are ignorant of the specific heat of steam at different pressures, yet we know the laws for other gases; we may therefore infer that, for an increase of pressure of two atmospheres, which we have supposed, the increase of temperature will undoubtedly be much greater than that of 212° to 275°.*

The law being general for specific heats of gases, which have been proved by experiment within small limits, we may consider ourselves warranted in deducing the existence of the law of partial condensation, otherwise the vapour of water is an anomaly among the other gases; it would therefore violate the law of continuity.

If the condensation were of great importance it would enter as an element into the calculation of the effect of engines working by expansion, as its effect is to accelerate the reduction of pressure arising from increase of volume, just as that effect would take place if the water of condensation were continually drawn from the cylinder as it was formed during the stroke. There are, besides the reduction of temperature, which is not taken into account in calculating the expansive force of steam, and from which there must necessarily follow an expansive force inferior to what is supposed, other causes of reduction in the effect of steam acting expansively. The object which we have had in view in what we have stated, has been simply to show that, in every case where the pressure of the steam is diminished, the temperatures corresponding to the successive pressures are always those which correspond to its formation under the given pressures as put down in the Table.

We shall enter more fully into this subject afterwards.

* A writer in the *Mechanics' Magazine* gives an investigation which shows that the "specific heats are inversely as the atomic weights," which is the celebrated result of the experiments of Dulong and Petit. A Table will afterwards be given which contains the results above-mentioned.

Table of the Properties of Steam, and of its useful Effect at Different Pressures.

Pressures.		Temperatures at the given pressures in degrees of Fahr.	Weight of a cubic foot of steam in lbs. avoirdupois.	Velocity of the steam into the atmosphere in English feet.	Theoretical horse-power of 1 lb. of Steam.								Pressure in lbs. per square inch above that of the atmosphere.
					Without condensation.				With condensation.				
In atmospheres.	In lbs. per square inch.				Without expansion.	Expansion at one-half.	Expansion at one-third.	Expansion at one-fourth.	Without expansion.	Expansion at one-half.	Expansion at one-third.	Expansion at one-fourth.	
1·00	14·70	212·00	0·0364	0	0·	-32·4	-95·2	-170·5	91·3	150·1	178·6	194·7	0
1·25	18·38	223·88	0·0447	873	21·5	10·1	-32·3	- 87·4	95·9	158·7	190·6	209·9	3·68
1·50	22·05	234·32	0·0529	1135	36·4	39·3	10·8	- 30·6	99·3	165·2	199·6	221·1	7·35
1·75	25·72	242·78	0·0609	1295	47·4	60·8	42·5	11·1	102·0	170·0	206·2	229·5	11·02
2·00	29·40	250·79	0·0688	1407	55·9	77·5	67·0	43·2	104·3	174·2	212·0	236·5	14·70
2·25	33·08	257·90	0·0766	1491	62·8	90·9	86·5	68·8	106·2	177·7	216·7	242·4	18·38
2·50	36·75	263·93	0·0844	1556	68·4	101·8	102·4	89·6	107·7	180·5	220·5	247·1	22·05
2·75	40·42	269·87	0·0921	1608	73·1	111·0	115·8	107·1	109·3	183·2	224·2	251·6	25·72
3·00	44·10	275·00	0·0998	1652	77·1	118·8	127·1	121·9	110·5	185·4	227·1	255·2	29·40
3·25	47·78	279·86	0·1073	1690	80·7	125·6	137·0	134·7	111·7	187·6	230·0	258·7	33·08
3·50	51·45	284·63	0·1148	1722	83·8	131·5	145·6	145·8	112·7	189·4	232·4	261·6	36·75
3·75	55·12	288·86	0·1223	1750	86·5	136·8	153·2	155·6	113·7	191·1	234·7	264·4	40·42
4·00	58·80	292·91	0·1298	1774	89·0	141·5	160·0	164·5	114·6	192·8	236·9	267·0	44·10
4·50	66·15	300·47	0·1445	1816	93·2	149·6	171·5	179·4	116·2	195·6	240·5	271·4	51·45
5·00	73·50	307·94	0·1590	1850	96·8	156·5	181·4	192·0	117·7	198·3	244·1	275·6	58·80
6·00	88·20	320·00	0·1878	1904	102·5	*167·2	196·5	211·4	120·2	202·6	249·7	282·2	73·50
7·00	102·90	331·56	0·2159	1945	107·0	175·6	208·4	226·5	122·4	206·4	254·6	288·1	88·20
8·00	117·60	341·83	0·2436	1978	110·6	182·4	217·9	238·4	124·3	209·7	258·8	293·1	102·90
9·00	132·30	351·32	0·2708	2006	113·7	188·2	225·9	248·5	126·0	212·8	262·7	297·6	117·60
10·00	147·00	359·60	0·2977	2029	116·3	193·0	232·5	256·7	127·5	215·3	266·0	301·4	132·30
12·50	183·75	377·42	0·3642	2074	121·5	202·5	245·5	273·0	130·7	220·8	272·9	309·5	169·05
15·00	220·50	392·90	0·4288	2109	125·7	210·0	255·6	285·4	133·4	225·5	278·9	316·4	205·80
17·50	257·25	406·40	0·4924	2136	129·0	216·0	263·6	295·2	135·7	229·5	283·9	322·2	242·55
20·00	294·00	418·46	0·5549	2159	131·8	221·0	270·3	303·3	137·8	233·0	288·3	327·2	279·30
25·00	367·50	439·34	0·6775	2196	136·3	229·1	281·0	316·2	141·2	238·9	295·7	335·8	352·80
30·00	441·00	457·16	0·7970	2226	140·0	235·6	289·5	326·4	144·2	244·0	302·0	343·1	426·30

The Indicator.

This highly useful little instrument was invented by Watt, to show the working condition of the steam engine as regards the pressure of steam in the cylinder, and the amount of vacuum or exhaustion attained by means of the air pump and condenser. Like the other productions of that master mind, it answers all the purposes for which it was intended, in the most complete and admirable manner. It is self-registering, and shows by means of a curve, which it describes on a slip of paper, the difference between the pressure of the atmosphere and the pressure within the cylinder of the engine, at every part of the up and down stroke of the piston, for that end of the cylinder to which it is attached.

It differs a little in form, but not in principle, and may be described generally as follows:—A small cylinder, perhaps never more than an inch nor less than a quarter of an inch in diameter, and from three to six inches long, having the top open to the atmosphere, and the bottom connected with the engine cylinder by means of a cock, which is generally inserted in the hole of the cylinder cover used for the grease cock. A piston is fitted into this cylinder so as to work free and easy, and, at the same time, steam and air tight. The rod of this piston is carried up into another small cylinder, around the inside of which is coiled a spiral spring fastened to the piston rod; this spring keeps the piston somewhere about half way down the cylinder when the pressures beneath and upon the indicator piston are in equilibrium; but when steam enters the engine cylinder, it passes through the cock and enters the indicator, and, if it be of greater pressure than the atmosphere, will raise the indicator piston and compress the spring. When exhaustion takes place in the engine cylinder, it will, of course, take place in the indicator too, and the atmosphere being of greater tension than the uncondensed vapour, will force down the indica-

tor piston and extend the spring. Now, as spiral springs, within practical bounds, are extended and compressed through equal spaces by equal and opposite pressures, and through double spaces by double pressures, and so forth, the amount of compression or extension of the spring will indicate the difference between the pressure under the indicator piston and the atmosphere; and to show this pressure in pounds per square inch, there is a longitudinal slot in the spring cylinder, through which passes a stud from the piston rod; on this stud is fixed an index-pointer which shows, on a scale fixed on the side of the slot, the pressure in pounds per square inch; of course the length of an unit of this scale is determined with reference to the stiffness of the spring and the diameter of the indicator piston. These are generally so proportioned that a tenth of an inch on the scale shall represent a pressure of one pound per square inch.

Suppose we wish to test the accuracy of an indicator: we may provide a balance beam or steelyard, having a weight at one end, and the piston rod of the indicator fastened to the other, the length of which is found by dividing the area of the indicator piston by the length of the arm to which the weight is to be attached; then if one pound, seven pounds, or fourteen pounds, cause the index to shew one unit, seven or fourteen units, on the scale, the indicator is right; if it show too much, the spring must be rejected, as too weak, or a new scale made; if it point to less, the spring must be "eased a little" with a file *between the coils, not on the outside,* until you find it right.

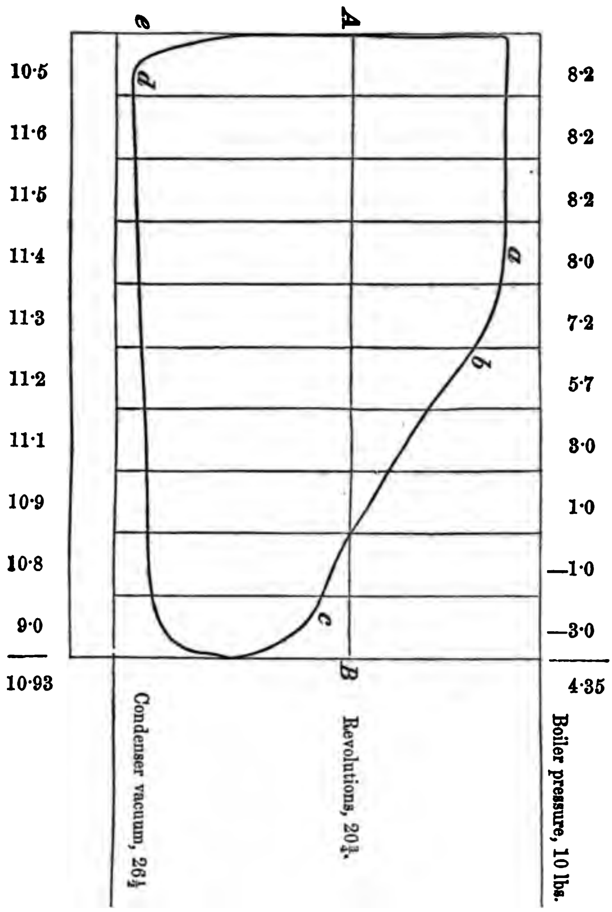
If the engine were to move slow enough, we might deliberately note the pressure by the scale at any part of the stroke of the engine; this not being the case, other parts are added, in order to make it self-registering. For this purpose another cylinder, round which the slip of paper is fastened, and which we will call the barrel, is placed alongside the other, so that a blacklead pencil fixed to the indicator piston rod may touch it. Within the

barrel is a spring which allows it to be turned nearly round, by a string passing round a pulley at the bottom of it to some convenient part of the engine, and, when the string is slacked, draws it back again, so that one-half of the barrel's motion is effected by the string, and the other half by means of the spring.

Suppose the barrel and its pulley to be six inches in circumference, the string passing over fairleader pulleys, and fastened to some part of the radius rod, having a motion of five inches, when the engine piston begins to descend, the barrel will begin to revolve, and when the piston gets to the bottom, the pencil will have marked a straight line five inches long on the slip of paper. When the piston begins to move upwards, the string will slack, and the spring will bring back the barrel. If we now open the cock, as the steam goes in and out, the indicator piston will rise and fall, and, if the barrel be at rest, the pencil will mark a straight line on the paper. The length of this line, measured by the scale, will give us the difference between the highest and lowest pressures of the stroke. If we now set the barrel in motion, we shall have the pencil moving up and down, while the paper is moving backwards and forwards, and the result of these combined motions will be a curve somewhat like *a b c d*.

When this has been done, the pencil must be lifted from the paper, the mechanism of its holder admitting of this, the cock must be shut, the steam let out from the indicator cylinder, and perfect equilibrium restored to its piston; the pencil is then again brought into contact with the paper, and the line A B is drawn to serve as a base, or zero, line. The operation of the instrument is now complete.

If the indicator cock had not been opened, the pencil would have traced the line A B. When the engine piston was at the top, the pencil would be at A; as the piston fell, the pencil would trace the line along towards B, arriving at B when the piston got to the bottom of its stroke, and retrace the line as the piston rose again to the top. But instead of the pencil being at A when the piston was at the



top of its stroke, we find it to be ·82 of an inch above A; this could only be in consequence of there being a pressure beneath the indicator piston greater than the pressure of the atmosphere upon it, and as a tenth of an inch represents a pound pressure on the square inch, we perceive that when the piston commenced its down stroke there was a pressure of 8·2 lbs. per square inch upon it. If this pressure had continued upon it until the piston got to the

bottom of the cylinder, we should have had a line drawn parallel to A B, and at the distance of ·82 of an inch from it; but we find this line only going about ·3 of the stroke, and at *a* it begins to bend downwards, whence we infer that the pressure remained so far at 8·2 lbs., and then began to decline, showing that the piston was moving away faster than the steam could be admitted—technically speaking, "the steam was being wiredrawn." At *b*, the nature of the curve changes, from being convex upwards to the reverse; this continues to *c*, where another change takes place. If we compare this curve with a theoretical curve, on the assumption that expansion according to Marriotte's law had gone on from *b* to *c*, we shall find them sufficiently similar to justify us in concluding that the valve was fully closed from *b* to *c*, and that expansion actually did go on between these two points. The rounding from *a* to *b* would be occasioned by the port becoming too contracted before the valve fully closed it. At *c* we find the pressure rapidly declining, and we conclude that the port has opened for exhaustion, and that the steam is rushing into the condenser. By the time the piston gets to the bottom of the cylinder, a partial vacuum is formed, which improves as the piston rises and nears the top, until it amounts to 11·6 lbs. at *d*, where another abrupt change takes place. Here we find the pressure of the atmosphere pressing upon the indicator piston is greater by 11·6 lbs. per square inch than the pressure in the cylinder; but from *d* to the end of the stroke this difference rapidly diminishes, and, as the pressure of the atmosphere must have remained constant, we have only two ways of accounting for the change—either fresh steam must have been admitted, or the steam yet remaining uncondensed must have been compressed. If steam had been admitted, it would have been steam of 8·2 lbs. pressure, and this meeting the piston coming up, would have sent the indicator piston up with a rapid motion, and the line would have gone up in nearly a perpendicular direction, crossing the line A B before the end of the stroke; we therefore conclude

that the uncondensed steam must have been compressed. The port, for about the last $\frac{1}{20}$ of the stroke, must have been closed. At the end of the stroke we find the 8·2 lbs. steam to have been admitted, and, as the slide must have been in motion during this time, we conclude that there must have been "lap" on the steam side of its face, and while the slide moved through the space of this lap the piston moved through $\frac{1}{20}$ of its stroke. If we allow $\frac{1}{20}$ also for clearance between the cylinder cover and the piston, we shall have steam of $15 - 11{\cdot}6 = 3{\cdot}4$ lbs. pressure; when the piston arrives at the top, this will be compressed into half its volume, and, therefore, be of 6·8 lbs. pressure, nearly corresponding with the diagram.

If we continue the curve *bc* at each end with a black-lead pencil, we may find the exact points where the steam port was fully closed by the valve, and where it opened for exhaustion, these being the two points of contrary fluxure, or where the lines we have drawn diverge from the curve drawn by the indicator pencil.

General Remarks.

When the card has been removed from the barrel, write the vessel's name upon it; if it be from a marine engine, the day of the month and time of day. On the face of it draw a line representing the perfect vacuum line, 15 lbs.; another representing the vacuum in the condenser, and one representing the pressure of steam at the time of the card being taken, and also the number of revolutions of the engine at the time; and write on the steam and condenser vacuum lines their figurative values. These lines serve as valuable checks, and as standards of comparison.

The diagram is then divided into ten parts by equidistant lines perpendicular to AB, and the value of the mean ordinate of each placed in the margin, for both steam and vacuum. That part of the steam line below AB being reckoned negative, and subtracted from the positive; the sums being divided by ten, we have the mean value of

steam and vacuum; these being added, we have the mean pressure on each square inch of the piston's surface.

These diagrams should always be taken from both the top and bottom of the cylinder, where it is convenient, *and for marine engines should also be taken for going astern*, as, in going into crowded harbours, it may be of the utmost consequence to have the engine go astern in the best possible manner, which can only be when the eccentric stops are properly placed, and which the indicator diagram will readily point out.

On Paddle Wheels.

*Action of the Paddle Wheels**.

Every paddle wheel may be regarded as a series of levers, coming successively into action. Each lever is represented by one of the arms of the wheels, to which a float is attached; the fulcrum is obtained by the reaction of the water on the paddle board; the resistance is that of the water to the motion of the vessel at the centre of the wheel, and in the direction of the vessel's course; and the power is that of the engine. Now, since the fulcrum is obtained by the reaction of the water on the float, and as there can be no reaction if the surface of the float does not move through the water, the true fulcrum must be at that point, on which, if immersed, there would be no reaction. This point moving in the direction of the surface of the float, can meet with no resistance from the water;

* When the paddle wheels of a vessel begin to revolve, the vessel itself being at rest, any point in one of the wheels describes a circle; but as the speed of the vessel increases, the circle becomes lengthened into a cycloid. First, it becomes what is called a curtate cycloid; then, at the time when the velocity of the vessel becomes equal to the circumferential velocity of the given point, it becomes a common cycloid; and if the speed of the vessel go on increasing, the point will describe the curve called the prolate cycloid.

some power, therefore, must be expended to force the floats through the water, in addition to that which would be required to propel the vessel.

The different Kinds of Paddle Wheel.

1. *Field's Paddle Wheel.*

This wheel, which, as well as others, is called the cycloidal wheel, differs from the common paddle wheel in the arrangement of the paddle boards; Mr. Galloway was the first who took out a patent for this invention, in 1835, but as Mr. Field employed a similar wheel two years previously, and exhibited a model of a cycloidal wheel before the Lords of the Admiralty, the credit of the invention, therefore, belongs to the latter gentleman.

Mr. Field thus describes his wheel, in the *London Journal* for December, 1835:—

" Each board is divided into several parts, or narrower boards, and arranged in, or nearly in, such cycloidal curves, that they all enter the water at the same place, in immediate succession, thus avoiding the shock produced by the entrance of the common board, so unpleasant to passengers, injurious to the vessel, and wasteful of the power. As the acting face of each board is radiating, it propels while passing under the centre in the ordinary way, and when it emerges, the water escapes simultaneously from each narrow board, and, consequently, cannot lop-up."

Mr. Barlow describes these wheels in the following manner:—"The principle of this contrivance consists in dividing the paddle into a number of parts, which are placed upon the wheel in the curve of a cycloid, so that they enter the water at the same spot, and follow one another so rapidly as to cause little resistance to the engine on entering the water, and afterwards separate, so as to afford full scope for their action in passing the centre,

and in coming out allow the water to escape readily from them."

With regard to the advantages and disadvantages of these wheels, there is some difference of opinion. Mr. Barlow says, that "its use is very likely to become general and supersede that of Morgan, from its superior strength and simplicity, while it does away with most of the evils to which the common wheel is subject."

Mr. Mornay, on the contrary, seems rather to underrate it. His opinion is, "that it remains to be decided by experience, whether the disadvantages of this wheel are overbalanced by the advantages which it *seems* to possess."

This wheel has been improved since the time of Mr. Galloway, in the following rather remarkable manner:— At the time of the adoption of this wheel, they were made with six or seven bars in each set, instead of the common paddle boards: this number has been much reduced; indeed, the wheels now used in Her Majesty's service have only two boards in a set, as every reduction was found to be more advantageous; however, these improvements cannot be carried on farther, as in that case they would return to the common wheel again.

Among the vessels which have been fitted with these wheels, are Her Majesty's steam vessels Tartarus, Rhadamanthus, Dee, and Meteor; the steam frigate Gorgon, of 1100 tons; the Great Western, and British Queen, American steamers; and the Hermes, a Government vessel of 730 tons.

2. *Paddle Wheels with Oblique Floats.*

These wheels will never become very much used, on account of some objections which I will mention presently.

One of the most simple is that of Mr. S. Hall, for which he obtained a patent in 1836. Here the paddle boards are placed obliquely to the rims and the axis of the wheel. The subject of the patent, however, is not the use of the oblique floats, but the making one-half of them

to enter the water in one diagonal direction, and the other in the reverse.

The objections are (1), that it requires a greater surface of paddle board to produce the same effect, for Mr. Mornay proves that the power in these wheels must exceed that in the common wheels in the ratio of $\frac{1}{10}$; and (2), that the shock is very little reduced, and the loss of power from oblique action is greater than with the common wheel.

3. *Perkins's Paddle Wheel.*

This wheel, for which Mr. Jacob Perkins took out a patent in 1829, differs from all others in the following things: the angle at which the floats are fixed to the shaft is 45°, and the shafts are carried in a sloping direction towards the stern, and meet in the plane of the keel. On the ends of the shafts farthest from the paddle wheels, are fixed bevel wheels, which act on each other, or are both acted upon by an intermediate bevel wheel in connection with the engine.

The disadvantges of this wheel are numerous; the large excess of the circumferential velocity of the floats over the speed of the vessel, the weight of the wheels compared with that of common wheels of equal effect, the use of bevel wheels to turn the shafts, and the inconvenient position of the wheels, which makes it difficult to support them, and exposes them to the winds and waves;—all these circumstances, together with the fact that it has not been adopted in practice, are conclusive of its disadvantages.

We next come to a totally different class of paddle wheels.

Paddle Wheels with Feathering Floats.

1. *Buchanan's Wheel.*

The construction of this wheel is as follows:—From the centre of a framing similar to that of the common wheel,

and called the pitch-wheel, are the radii, or arms; the spindles, which work in bearings in the circumference of the pitch-wheel, are attached to a float, which is divided into two equal parts by the spindle, one end of which passes through the pitch-wheel, and is furnished with a lever. The extremity of this lever is bent at right-angles to it, and works in a bearing on the circumference of a flat wheel or frame called the connection-wheel, which turns on an axle fitted to the ship's side, having its centre at a different point. This axle is made sufficiently large to admit of the shaft passing through it.

In this paddle wheel, taking the same data as he had for the common wheel, Mr. Mornay calculated that in the common wheel the proportion of the effective to the total power :: 0·582 : 1; whereas in this, the ratio is merely as 0·443 : 1; as in the steam frigate Salamander, with the common wheels, the whole horse power being 151·66, the effective will be 88·23. In Buchanan's, the power required to produce the same effect will be 199·31.

2. *Oldham's Wheel.*

The principle of this wheel is almost the same as if the axis of the eccentric in Buchanan's moved round the main shaft in the same direction as the latter, and with half its angular velocity. This is effected by spur-wheels placed on different shafts, working in each other, and of different dimensions. The action of this wheel is theoretically very good, as the pressure on the floats is nearly uniform throughout the stroke; but in a practical view the friction, the liability to get out of order, and the other difficulties which attend it, overbalance the theoretical advantages, and render it so exceptionable that it can never succeed, unless some better method of carrying the principle into execution be invented. It is, however, of little advantage to attempt the improvement of this wheel, as the following, besides being simple in construction, possesses the theoretical advantages in a higher degree.

3. *Morgan's Paddle Wheel.*

The construction of this wheel is as follows:—The paddles turn on spindles, having a bearing on the framework and on the wheel, which is polygonal, having as many sides as there are paddles. The inside frame or polygon alone is attached to the shaft of the engine, which does not continue beyond the side of the vessel; and the outer one has an independent bearing on a centre attached to the paddle-box, so that it derives its motion entirely from the arms or angles of the polygon; the space between the two frames of the wheels being left quite free. A crank is fixed to the paddle-box, on which the outer polygon revolves; it projects in an inclined direction in the open space between the sides of the wheel, but to a point considerably eccentric with it. Each paddle has a crank attached to it at an angle of about 70°, and arms connect the extremity of the cranks with a movable boss, which revolves upon a fixed centre. One of these arms is fixed to the boss, and is called the dividing arm.

This wheel is quite free from back water, and from any shock, as both the upper and lower edges of the float are nearly tangents to their respective cycloids at the time of entering the water. These wheels have always been praised for their beautiful action, their strength, and their durability, as well as for the safety, comfort, and economy with which they are attended. They have been adopted in many government steamers, and always with success. It has been found by experiments, that in light immersions little or no advantage is gained by these wheels, since, reckoning the power of the engine 1, the effective power $= \cdot 660$ in the common wheel, and $\cdot 666$ in Morgan's. This arises from the loss from the additional velocity required to obtain the resistance or receding of the vessel being fully equal to that of oblique action in the common wheel. In cases of deep immersion the case is different; the effective power of the common wheel will,

in a very deep immersion, equal only ·553, while that of Morgan's wheel remains the same.

In the Firebrand steamer, Morgan's wheel, with less power and less float, gave a greater speed than the common wheel. The economy is also to be noticed in Mr. Barlow's Memoir.

We may, therefore, conclude that Morgan's paddle wheel has the superiority in sea, and the common wheel in river navigation.

Centre of Pressure.

To find the centre of pressure of a paddle-board, is, taking everything into account, a problem of the most difficult kind; but for all practical purposes the following rule, made from Mr. Barlow's formula, is sufficiently exact.

To find the centre of pressure in the common paddle-wheel.

Rule.—To the difference between the radius of the rolling circle and that of the wheel, add the depth of the paddle; divide the fourth power of this sum by four times the depth of the paddle, and find the cube root of the quotient: from this root subtract the above-named difference between the radius of the rolling circle and that of the wheel; the remainder is the distance of the centre of pressure from the upper edge of the paddle-board.

Example.—The diameter of the wheel of the Messenger steam vessel is 19 feet 4 inches, the depth of the paddle-board is 2 feet, the diameter of the rolling circle is 13·31 feet; find the diameter of the centre of pressure.

The radius of the wheel is 9 feet 8 inches or 9·666 feet, the radius of the rolling circle is 6·655, the difference of these radii is 3·011, then $3·011 + 2 = 5·011$, the fourth power of which is 630·52; this, divided by four times the depth of the paddle, gives $\frac{630·52}{8} = 78·82$ nearly: the

cube root of this is 4·287, and $4·287 - 3·011 = 1·276 =$ the distance of the centre of pressure from the upper side of the paddle; now, as the radius of the wheel is 9·666, and the depth of the paddle being 2 feet, the distance from the centre of the wheel to the upper side of the paddle is 7·666. Then $7·666 + 1·276 = 8·972$, which is the distance from the centre of the wheel to the centre of pressure, and $8·972 \times 2 = 17·944$ for the diameter to the centre of pressure.

Since the velocity of the rolling circle is equal to the velocity of the vessel, we can easily find the diameter of that circle.

Suppose the speed of a vessel was found to be 9·75 English miles per hour. The number of feet in one mile is 5280, therefore, $9·75 \times 5280 = 51480$ feet per hour, and 51480 divided by 60 gives 858 feet per minute: the number of strokes per minute of this vessel is 22; hence, 858 divided by 22, gives 39 for the circumference of the rolling circle, and 39 divided by $3·1416 = 12·4$ feet, the diameter of the rolling circle.

The diameter of the centre of pressure, or effective diameter of the wheel, being known, we at once attain the velocity of the wheel in excess of that of the vessel, or that at which it recedes in the water to produce the resistance necessary for propelling the vessel. The rule for ascertaining the amount of this resistance or pressure on the vertical paddle, is to multiply the square of this velocity by the area of the paddle-board and by $62\frac{1}{2}$ (the weight of a cubic foot of water in pounds), and dividing by $64\frac{1}{3}$, the pressure upon a surface moving in a fluid being equal to the weight of a column of water whose base is the area of the surface, and the altitude equal to the space, a heavy body must fall through to acquire the velocity. This number, multiplied by the velocity of the wheel, will express the power expended on the vertical paddles; and this, divided by the whole power of the engine, will give the proportion consumed on the vertical paddle.

The diameter of the centre of pressure being 17·944 feet,

its circumference is $17{\cdot}944 \times 3{\cdot}1416 = 56{\cdot}37287$, and this multiplied by 22, the number of strokes per minute, gives 1240·4 for the velocity of the centre of pressure in feet per minute, or 20·67 for the velocity in feet per second.

The velocity of the rolling circle, from above, is 858 feet per minute, which is 14·3 feet per second; hence $20{\cdot}67 - 14{\cdot}3 = 6{\cdot}37$ feet per second, which is the excess of the velocity of the wheel over that of the vessel, and $(6{\cdot}37)^2 \times$ area of paddle-board $\times \dfrac{62\frac{1}{2}}{64\frac{1}{3}} =$ pressure in pounds on the vertical paddle.

The paddle-board of the above-named vessel was 10 feet long and 2 feet broad, hence the area is 20 square feet;

$$\text{then } \frac{(6{\cdot}37)^2 \times 20 \times 62\frac{1}{2}}{64\frac{1}{3}} = \frac{50723{\cdot}125}{64{\cdot}334} = 788{\cdot}4;$$

on both paddles it is 1576·8.

The velocity of the centre of pressure is 1240·2 feet per minute;

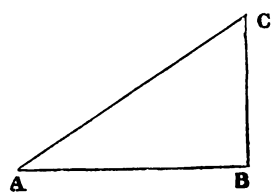
$$\therefore \frac{1240{\cdot}2 \times 1576{\cdot}8}{33000} = 59 \text{ horse-power.}$$

Screws.

Screws are of various kinds, according to pitch, number of threads, and form of thread. If we take a piece of wire having a triangular section, and wind it round a cylinder, with one of its sides next the cylinder, we shall have a representation of what a mechanic would call a V-threaded screw, of a single thread. The pitch would be the distance between two adjacent threads measured in the direction of the axis. For instance, if the base of the V were a quarter of an inch, the pitch would be a quarter of an inch nearly. The screw would then resemble those used in machinery. If we coil another such wire round the inside of a cylinder, with the base of the V touching the cylinder, we shall then have a resemblance of the common "nut" so well known to Mechanics. If we

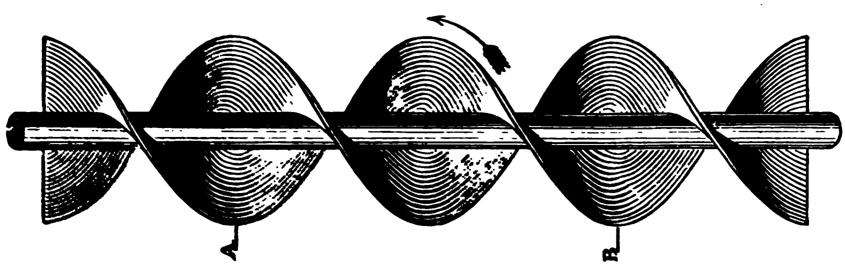
fit the screw into the nut and turn it once round, we shall find that it will advance through the nut one-quarter of an inch; we may, hence, define the pitch of a screw to be—the amount it would advance when turned once round in a fixed nut. If two such wires were wound round a cylinder, we should have a two-threaded screw, of twice the pitch of the former, and, generally, n wires would give an n-threaded screw of n times the pitch. These remarks may be very simply illustrated by winding a piece of common twine round a smooth walking-stick. If the twine be a tenth of an inch in diameter, we shall have a screw of a tenth of an inch pitch. If we wind two strings round, both touching the cylinder, we shall have a two-threaded screw of a fifth of an inch pitch. If the strings be fastened at each end, we may turn the stick round in a nut formed by the hand, and notice the advance of the stick or pitch of the screw. If, when two or more strings have been used, we take all off except one, we shall have a single-threaded screw of the same pitch as when the whole of the threads were used, and we shall see, that to measure the pitch of a screw we must measure from the centre of the thread to the centre of the *same thread* after it has passed once round. If the thread be not regular, but more inclined to the axis at one part than at another, the thread would be pronounced by an engineer to be "drunk:" inexperienced workmen in "striking threads by hand," in the lathe, sometimes produce these drunken screws. A practised eye, on seeing such a screw running round in a lathe, will detect a very small amount of *inebriety*. They also sometimes strike a double thread instead of a single one, and get exceedingly puzzled on finding that the nut will not go on to it.

A screw may also be represented by a triangle wound round a cylinder. If we cut out of a paper a right-angled triangle, as in the figure, and wind it round a cylinder, the circumference of



which is equal to A B, then A C will represent the thread of a screw, and B C will be the pitch of it, when wound round so as to bring the point A to touch at B.

The screws used as propellers for steam vessels are generally two-threaded, sometimes three, and rarely four, and seldom much more than one-sixth of the pitch in length. The thread is thin, and stands a great way out from the cylinder, or "boss," round which it passes something like that in the annexed figure.



Let it be required to find the pitch of a screw to make 60 revolutions per minute, the vessel expected to go 10 knots per hour, and the screw to "slip" 15 per cent.; *i. e.* while the screw goes 100 knots the ship shall only go 85.

When the ship goes 10 knots, the screw must go

$$10 \times \frac{100}{85} = 11{\cdot}764.$$

Now, one knot is equal to 6080 feet;

$$\text{hence, } 6080 \times 11{\cdot}764 = 71525{\cdot}12 \text{ feet per hour,}$$

$$\text{and } \frac{71525{\cdot}12}{60} = 1192{\cdot}085 \text{ feet per minute;}$$

this divided by 60, the number of revolutions, gives 9·868 feet as the required pitch.

	Diameter of Cylinder.	Stroke.	Revolutions per Minute.	Speed of Vessels in knots of 6080 ft. per hour.	Horse power.		Paddle-wheel.		Paddle-floats.			
					Nominal.	Actual.	Diameter.	Dip.	Length.	Depth.	No.	Kind.
	Inches.	ft. in.	No.	Knots.			ft. in.	ft. in.	ft. in.	ft. in.		
La Plata.........	103½	9 0	16½	13·016	876·66	2548·8	36 2	5 0	10 6	3 3	28	Common.
Arabia	103	9 0	18	14	873	3256	36 0	5 3	10 7	3 2	28	
Bogota	73	6 0	20	12	394	1531·8	27 6	4 1	8 6	2 8	20	
Lima	...	...	24	13	...	1428	26 0	3 9	8 0	3 0	...	
Quito	...	...	21½	13	...	1298	26 9	4 1	8 6	2 8	...	
Albion	44	3 6	33	11·368	120	407·65	17 3	2 0	8 6	1 6	16	
Orinoco	2 No. 69	9 0	13½	11·845	787	1939·7	40 0	8 6	11 6	4 6	15	Feathering.
Amazon	96	...	14	11·495	761·6	2073·3	39 0	8 9	12 0	...	16	
Parana	96	...	15	12·5	...	2265·9	39 6	6 9	...	...	16	

	Diameter of Cylinder.	Stroke.	Revolutions per Minute.	Speed of Vessels in knots of 6080 ft. per hour.	Nominal.	Actual.	Screw Propeller.				Multiple of Gearing.
							Diameter.	Length.	Pitch.	Blades.	
							ft. in.	ft. in.	ft. in.	No.	
Shanghai	39	2 0	94	10·31	100	396	9 0	2 7	15 6	2	1
Chusan	...	...	93	9·25	...	363·2	9 8	...	...	3	1
Argus...........	= 24¼ 28—14	1 4	110	8·54	60	149·59	8 0	1 7	9 6	2	1
Bombay	63½	5 0	25½	11·025	282·3	663·88	14 0	2 1	18 to 24 ft.	3	2½
Sydney	66	4 6	29¼	10·574	300	939·9	14 0	3 4	18 to 19½ ft.	2	2¼
Hydaspes	55	3 0	54	9·08	...	699·1	15 6	3 0	20 ft.	2	1

The following letter appeared in the "Nautical Standard" last March.

Negative Slip in Screws.

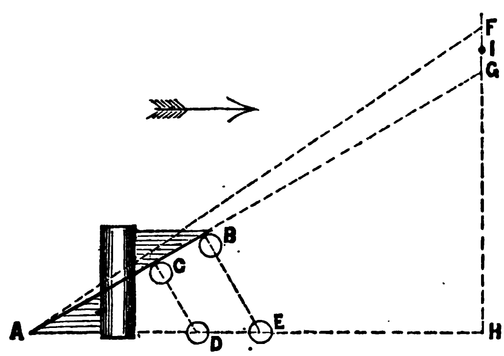
To the Editor of the Nautical Standard.

Sir,—The negative slip of the screw, as it is called, appears to be engaging at present the attention of the nautical part of the engineering world. I have had many applications for an explanation of this anomaly. The only rational way, in my opinion, of explaining it, is that which was given to me in 1849, by my able and talented friend, Mr. Holland, one of the Admiralty engineers. It certainly appears to be something more than a paradox for a vessel to be propelled, by any power whatever, faster than the power is itself moving. A writer in the "Artizan" has pertinently observed, that it is just as absurd as to suppose that a barrow should go faster than the man that is wheeling it. Mr. Holland takes a more philosophical view of the subject. He considers that the force of the water acting on the outer part of the screw, tends to spring it so as to alter its pitch; and this alteration in the pitch of the screw is quite sufficient to account for the apparent negative slip; and nothing, in my opinion, can be more clear, than that the screw will be in some degree sprung by its action on the water. The following is Mr. Holland's explanation:—

If A B represent the edge of a screw-blade, and A H G the triangle, which, when wound round a cylinder, would describe the screw, then G H will represent the pitch. Now, when the screw turns in the direction of the arrow, a particle of water met with at B, will be driven to somewhere about E, when the vessel is at her moorings; another particle met with at C, will offer a decreasing resistance to the blade from C to A, and be left somewhere about D. The part of the blade B will strike against water having little or no motion; whereas, the after-part of the blade will strike against water having a consider-

able amount of motion, and will, therefore, meet with less resistance than the fore part; the effect of this will be to twist the blade and increase the pitch. If the vessel be attached to a dynamometer, and we calculate the effect without allowing for this increase of pitch, we shall arrive at an erroneous result.

This increase of pitch must also take place when the vessel is under weigh, and will explain that strange anomaly "negative slip;" for if, while the screw goes one turn, the vessel goes the distance I H, we have a negative slip I G, but if we suppose the screw-blade to "spring" to the dotted line F A, we then have a positive slip F I.



To show that the power cannot move slower than the vessel, let us take the paddle-wheel, and let us suppose, if it were possible, that the velocity of the wheel was exactly equal to the velocity of the vessel, then, evidently, the paddles would be relatively at rest, and could have no propelling power. If they moved slower, it is equally evident that the paddles would retard the motion of the vessel. Now, if this be true, how is it possible that any vessel can move faster than the power that is moving it?

As regards the pitch connected with the negative slip of the screw, we shall take the Plumper steam-vessel. On December 1st, 1848, in Stokes's Bay, Portsmouth, the speed of the screw in knots per hour was 6·188, and the

speed of the vessel was 6·497, which would show a negative slip of ·329. The diameter of the screw is 9 feet; the pitch, 5 ft. 7 in.; length, 1 ft. Revolutions, 11·2 per minute.

Now, if we suppose that, instead of the pitch remaining at 5 ft. 7 in., the screw is sprung so as to be 6 ft. pitch,

Then $112 \times 6 \times 60 = 40320$ feet per hour.

$$\text{And } \frac{40320}{6080} = 6{\cdot}631 \text{ knots per hour,}$$

which shows a *positive* slip of ·134.

From the various enquiries I have made of practical men, I find that, in screws of great stiffness, no negative slip is observed; but where the screws are comparatively slight, there appears to be a negative slip; in fact, it has been ascertained that some vessels have had the screws so slightly made that they have actually broken, which shows that previously to breaking the pitch must have increased, and would therefore indicate negative slip.

On so important a subject as the screw, too many observations and experiments cannot be made, so as not only to set the question of negative slip at rest, but many other questions which seem not as yet to have been fully elucidated.

Mr. Holland is a person well versed both in theory and practice, and possesses keen observing powers, therefore his opinion—to say the least—is worthy of respect; but I am quite sure that he is open to conviction, if any one can show him that his views are fallacious. He would rather court inquiry than shrink from it, and it has been beautifully observed by one of our English writers, that "inquiry is to truth what friction is to the diamond: it proves its hardness, adds to its lustre, and excites new admiration."

I am, Sir,

Your obedient Servant,

JAMES HANN.

King's College, March 4, 1853.

Professor Woodcroft was the first to introduce the screw blade with a rising pitch, that is, with a pitch which varies at every point along the extremity of the screw blade.

He also has the merit of being the first to introduce another variation of considerable importance, which is to turn the screw blades through an angle in opposite directions. This variation, which is an admirable expedient to overcome a great difficulty, he thinks will produce an effect in the velocity of the vessel, somewhat similar to that produced by increasing or diminishing the pitch in the constant pitch surface.

This variation is not useful in the screw propeller with a rising pitch only, but also in the one with a constant pitch. Suppose the blade of the screw to have its extreme helix making an angle of 20°, and Professor Woodcroft's apparatus to be applied in such a manner as to turn this helix through every angle to 15°; then, if we could persuade ourselves that the screw blade thus moved from its first position to its second fully coincided with a screw blade whose angle is 15°, the variation to which we allude would be a great improvement in the art of screw propulsion. If the angle through which this variation takes place be small, the screw blade, in its second position, will nearly coincide with a screw blade having the same angle; but generally this coincidence does not take place, and the relation of the screw blade to the axis of rotation in the one case is not the same as in the other.

In the screw the pitch is proportional to the tangent of the angle of the screw.

Rule.—Multiply the tangent of the angle of the screw by 3·1416 and by the diameter of the cylinder; the result is the pitch.

Example.—Given the radius of the cylinder 8 feet, the angle of the screw 17° 30′, to find the pitch.

Then, by the rule,

$$2 \times 3{\cdot}14156 \times 8 \times \tan 17^{\circ}{\cdot}50$$

$$= 50{\cdot}26496 \times \tan 17^{\circ}{\cdot}50.$$

Now, tan $17°·30 = ·3152988$;

$$\therefore\ 50·26496 \times ·3152988 = 15·85 \text{ nearly.}$$

In the same ship and the same screws, the horses' power varies as the cube of the velocity of the ship.

In the same ship and different screws, the horses' power varies as the rectangle of the pitch and square of the velocity.

On Winding Engines.

In winding engines drawing coals out of a pit, where we intend them to go a given number of strokes in drawing a corf, we must ascertain the diameter of the roll at first lift. In this case, we suppose the engine to have flat ropes, such as are generally used, and which lie upon each other.

To find the diameter of a rope roll at the first lift, it is necessary to know the depth of the pit, the thickness of the rope, and the number of strokes which you intend the engine to make in drawing up a corf or corves.

Then, the thickness of the rope being known, and the number of strokes, we can determine the thickness of rope upon the roll, let the diameter of the roll be what it may. Thus, suppose the thickness of the rope to be 1 inch, and the number of strokes 10; then the radius of the roll is increased 10 inches, or the diameter is increased 20 inches, whatever that diameter may be.

Rule.—Multiply the depth of the pit, in inches, by the thickness of the rope, also in inches, for a dividend.

Then multiply 3·1416 times the thickness of the rope, in inches, by the number of strokes, for a divisor.

Divide the above dividend by this divisor, and from the quotient subtract the product, which is found by multiplying the thickness of the rope by the number of strokes, and the remainder will give the diameter of the roll in inches.

Example.—If an engine makes 20 strokes in drawing a corf up a pit, the depth of which is 100 fathoms, and the thickness of the rope 1 inch, what is the diameter of the roll at the first lift?

$$100 \text{ fathoms} = 7200 \text{ inches, and } 7200 \times 1 = 7200,$$

which is the dividend mentioned in the rule.

And $3{\cdot}1416 \times 1 \times 20 = 62{\cdot}832$, the divisor mentioned in the rule.

$$7200 \div 62{\cdot}832 = 112{\cdot}8 \text{ nearly,}$$

$$\text{and } 112{\cdot}8 - 20 = 92{\cdot}8 \text{ inches} = 7 \text{ feet } 8\tfrac{8}{10} \text{ inches.}$$

It may be remarked here, that if an engine be drawing coals out of a pit with round ropes, and we wish to take the round ropes off and to put flat ones on, this rule will determine what diameter our roll must be at first lift, so that the engine may go the same number of strokes as before, when the round ropes were on.

Example.—If an engine goes 10 strokes in drawing a corf up a pit, the depth of which is 60 fathoms, with round ropes, where the round ropes do not lie upon each other, what must be the diameter of a flat rope roll, so that the engine may go the same number of strokes as before, the thickness of the rope being half an inch?

$$60 \text{ fathoms} = 4320 \text{ inches,}$$

$4320 \times \frac{1}{2} = 2160$, the dividend mentioned in the rule;

$$3{\cdot}1416 \times \tfrac{1}{2} \times 10 = 15{\cdot}708,$$

the divisor which is mentioned in the rule;

$$2160 \div 15{\cdot}708 = 137{\cdot}5.$$

And the product of the thickness of the rope and number of strokes is $\frac{1}{2} \times 10 = 5$.

Hence $137{\cdot}5 - 5 = 132{\cdot}5$ inches $= 11$ feet $0\frac{1}{2}$ inch.

When an engine draws coals out of a pit, with flat ropes, the corves will not pass each other at mid-shaft, that is, half way between the top and bottom of the pit; for the corf which goes from the top of the pit will pass

through a greater space in the same time than the corf which comes from the bottom, owing to the circumference of the roll being always greater until the engine has performed half her number of strokes. Therefore, meetings will always be below mid-shaft. At meetings the rolls are equal; and, after this, the roll on which the ascending corf hangs continues to increase until the corf arrives at the top of the pit.

When the depth of the pit, the thickness of the rope, and the diameter of the roll are given, to find where the corves will meet, we have the following

Rule.—Multiply the radius of the roll by the depth of the pit. Call the result the numerator of the first fraction.

Multiply the depth of the pit by the thickness of the rope, and divide the product by 3·1416. Add this quotient to the square of the radius of the roll, and extract the square root of the sum. Add this square root to the radius of the roll. Call the result the denominator of the first fraction.

Multiply the thickness of the rope by the square of the depth of the pit. Call the result the numerator of the second fraction.

Square the denominator of the first fraction before found, and multiply that square by 4 times 3·1416. Call the result the denominator of the second fraction.

Add the two fractions, and this gives the distance where the corves will meet.

Example.—Whereabouts in a coal shaft will the corves meet, if the radius of the roll be $3\frac{1}{2}$ feet, the thickness of the rope $\frac{1}{8}$ foot, and the depth of the pit 1020 feet?

By the rule—

$$3\tfrac{1}{2} \times 1020 = 3570$$

$$\frac{1020 \times \frac{1}{8}}{3{\cdot}1416} = 40{\cdot}58441558;$$

$$3{\cdot}5^2 = 12{\cdot}25$$

$$12{\cdot}25 + 40{\cdot}58441558 = 52{\cdot}83441558;$$

$$\sqrt{52 \cdot 83441558} = 7 \cdot 2687;$$

$$3 \cdot 5 + 7 \cdot 2687 = 10 \cdot 7687;$$

hence $\dfrac{3570}{10 \cdot 7687}$ is the first fraction.

Again, $\frac{1}{8} \times 1020^2 = 130050;$

$$10 \cdot 7686^2 \times 4 \times 3 \cdot 1416 = 115 \cdot 96489969 \times 12 \cdot 5664$$
$$= 1457 \cdot 261315;$$

hence $\dfrac{130050}{1457 \cdot 261315}$ is the second fraction.

$$\frac{3570}{10 \cdot 7687} + \frac{130050}{1457 \cdot 261315} = 331 \cdot 51634 + 89 \cdot 24274$$

$$= 420 \cdot 75908 \text{ feet} = 70 \cdot 1265 \text{ fathoms.}$$

THEORETICAL INVESTIGATION.

If s represent the number of feet described at any part of the stroke, p' the pressure when that part is described, a the number of feet described before the steam is cut off, and l the length of the stroke in feet, then, p being the pressure at which the steam is admitted, we have, by Mariotte's law,

$$p' : p :: a : s,$$

$$\text{or } p' = \frac{p\,a}{s}.$$

Therefore, the variable work $\int p'ds$ taken between the proper limits is

$$\int_a^l \frac{p a d s}{s} = ap \int_a^l \frac{ds}{s} = ap\,(\log l - \log a)$$

$$= ap \log\left(\frac{l}{a}\right) *,$$

this is the work done per square inch during expansion.

The work done before expansion is evidently represented by ap.

Hence, the whole work done per square inch is

$$ap + ap \,.\log\left(\frac{l}{a}\right). \quad \text{(Rule 1, page 3.)}$$

If $L =$ load per square inch, then Ll is the work ex-

* The logarithms here used are the hyperbolic.

erted upon the load each stroke. Now, as the work of the steam must be equal to the work done upon the load, we have

$$L \, . \, l = a p \left\{1 + \log . \left(\frac{l}{a}\right)\right\},$$

$$\therefore \; L = \frac{ap}{l} \cdot \left\{1 + \log . \left(\frac{l}{a}\right)\right\}. \text{ (Rule 2, page 3.)}$$

From the above formula, we evidently have

$$p = \frac{L \, . \, l}{a\left\{1 + \log . \left(\frac{l}{a}\right)\right\}}. \text{ (Rule 3, page 3.)}$$

To find the velocity of the piston when any part of the stroke is described, we must apply the principle of *vis viva**. It is demonstrated by Poncelet, in the MECANIQUE INDUSTRIELLE, and by Moseley, in the PRINCIPLES OF ENGINEERING, *that the work accumulated in any body or machine is half the vis viva*†; so that if U be the work

* The *vis viva* of a body is its mass multiplied by the square of its velocity.

† For, suppose a given weight W to descend freely by gravity from a height H, we know that if from the point to which the body has descended, it were projected upwards with the velocity it has acquired at that point, it would ascend to the same height H, from which it fell to acquire that velocity; and would, therefore, perform an amount of work represented by WH units, so that in its descent it has, in fact, accumulated that amount of work, viz. WH units. Now, H being the height from which the body has descended, if V represent the velocity acquired in its descent, we have

$$V^2 = 2 g H,$$

$$\text{hence, } H = \frac{1}{2} \cdot \frac{V^2}{g};$$

$$\text{therefore, the work accumulated} = W \frac{V^2}{2g} = \frac{1}{2} \frac{W}{g} V^2;$$

but $\frac{W}{g}$ is the mass of the body and is represented by M; hence

$$\text{the work accumulated} = \frac{1}{2} M V^2 = \frac{1}{2} \textit{ vis viva}.$$

Hann's Mechanics, p. 129.

done upon any body whosé weight is W, and through any space s, and U' the work expended upon the resistance which opposes the motion of the body through the same space, then $U - U'$ is the work accumulated in the body. Now, if V represent the velocity, and g the accelerative force of gravity, the *vis viva* is $\frac{W}{g} \cdot V^2$; the accumulated work is, therefore,

$$\frac{1}{2} \cdot \frac{W}{g} \cdot V^2 = U - U'.$$

Suppose the whole mass to move with the same velocity as the piston; the work done upon each square inch by the steam, when s feet of the stroke have been described, is

$$pa\left(1 + \log\frac{s}{a}\right),$$

and the work expended on the load up to that point is

$$L \cdot s;$$

hence the work accumulated upon each square inch is represented by

$$pa\left(1 + \log\frac{s}{a}\right) - Ls,$$

$$\text{or } \pi r^2 \left\{pa\left(1 + \log\frac{s}{a}\right) - Ls\right\} = U - U',$$

which is the accumulated work on the whole piston, r being the radius of the piston;

$$\therefore \frac{1}{2} \cdot \frac{W}{g} \cdot V^2 = \pi r^2 \left\{pa\left(1 + \log\frac{s}{a}\right) - L \cdot s\right\},$$

$$\text{and } V^2 = \frac{2g\pi r^2}{W}\left\{pa\left(1 + \log\frac{s}{a}\right) - L \cdot s\right\},$$

substituting for L, its value $\frac{pa}{l}\left(1 + \log \cdot \frac{l}{a}\right)$,

$$V^2 = \frac{2g\pi r^2}{W}\left\{pa\left(1 + \log.\frac{s}{a}\right) - \frac{pas}{l}.\left(1 + \log\frac{l}{a}\right)\right\}$$

$$= 2g\pi.\frac{par^2}{W}.\left\{1 + \log\frac{s}{a} - \frac{s}{l}\left(1 + \log.\frac{l}{a}\right)\right\}.$$

To find when the velocity of the piston is a maximum, the expression

$$\frac{2g\pi.par^2}{W.}\left\{1 + \log.\frac{s}{a} - \frac{s}{l}\left(1 + \log.\frac{l}{a}\right)\right\}$$

must be a maximum;

hence $\left\{1 + \log.\frac{s}{a} - \frac{s}{l}\left(1 + \log.\frac{l}{a}\right)\right\}$ is a maximum;

or, $1 + \log.s - \log.a - \frac{s}{l}\left(1 + \log.\frac{l}{a}\right) =$ maximum.

If $u = 1 + \log.s - \log.a - \frac{s}{l}\left(1 + \log.\frac{l}{a}\right)$

$$du = \frac{ds}{s} - \frac{ds}{l}\left(1 + \log.\frac{l}{a}\right) = 0$$

$$\frac{du}{ds} = \frac{1}{s} - \frac{1}{l}\left(1 + \log.\frac{l}{a}\right) = 0,$$

$$\therefore\ s = \frac{l}{1 + \log.\frac{l}{a}}.\quad \text{(Rule 4, page 4.)}$$

$\frac{d^2u}{ds^2}$ is evidently negative, indicating the maximum; therefore, the value of s substituted in the expression for the velocity, gives

$$V^2 = 2g\pi.\frac{par^2}{W}\left\{1 + \log.\frac{l}{a\left(1 + \log.\frac{l}{a}\right)} - \frac{1}{1 + \log.\frac{l}{a}}.\left(1 + \log.\frac{l}{a}\right)\right\}$$

$$= 2g\pi \,.\, \frac{par^2}{W} \left\{ \log \,.\, \frac{l}{a\left(1 + \log \,.\, \frac{l}{a}\right)} \right\}$$

for the greatest velocity.

This result can be obtained without the aid of the Differential Calculus, in the following manner:—

Let x be the distance in feet the piston has moved to acquire its greatest velocity; then, by Mariotte's law,

$$\frac{x}{a} = \frac{p}{P}, \quad \text{or,} \quad x = \frac{ap}{P};$$

but, in accelerated motion, the velocity is greatest when the resistance becomes equal to the accelerating force; hence, in this case, the velocity is greatest when the pressure is equal to the load, or when

$$x = \frac{ap}{P} \text{ becomes } x = \frac{ap}{L};$$

substitute this value of x for s in the equation for the velocity, and we have

$$V^2 = \frac{2g\pi par^2}{W} \left\{ 1 + \log \,.\, \frac{\frac{pa}{L}}{a} - \frac{\frac{pa}{L}}{l}\left(1 + \log \,.\, \frac{l}{a}\right) \right\}$$

$$= \frac{2g\pi par^2}{W} \,.\, \left\{ 1 + \log \,.\, \frac{p}{L} - \frac{pa}{Ll}\left(1 + \log \,.\, \frac{l}{a}\right) \right\}$$

$$= \frac{2g\pi \,.\, par^2}{W} \left\{ \log \frac{l}{a\left(1 + \log \,.\, \frac{l}{a}\right)} \right\} \text{ by substituting}$$

for L its value $\quad \frac{pa}{l}\left(1 + \log \,.\, \frac{l}{a}\right)$.

Pambour, in his excellent work on the *Theory of the Steam Engine,* grounds that theory on the two following principles:—

First, *that the engine having attained uniform motion, there is necessarily an equilibrium between the pressure*

of the steam in the cylinder and the resistance against the piston, that is,

$$p = R \quad \text{..................} \quad (1),$$

R being the whole resistance against the piston.

Secondly, that *there is necessarily an equality between the production of steam and its expenditure.*

If P is the pressure of steam in the boiler, A the area of the piston, v the velocity of the piston in an unit of time, S the volume of water evaporated in an unit of time, m the ratio of the volume of steam formed under the pressure P of the boiler to the volume of water that has produced it; then mS will be the volume of steam formed in the same unit of time and under the pressure P in the boiler, and supposing, according to Mariotte's law, that the temperature remains the same, the volume mS of steam supplied each unit of time by the boiler when transmitted to the cylinder, will become $mS \cdot \frac{P}{p}$; and $A \cdot v$ being the *expenditure* of *steam*, we can express the second principle as follows:—

$$Av = mS \cdot \frac{P}{p} \quad \text{..........................} \quad (2).$$

But by equation (1), $p = R$,

$$\therefore A \cdot v = mS \cdot \frac{P}{R} \quad \text{..............................} \quad (3).$$

If the unit of time be one minute, and N the number of strokes in that time, l being the length of the stroke and a the number of feet described before the steam is cut off, then

$$v = Nl$$

$$\therefore N = \frac{v}{l};$$

and the expenditure of steam per minute being

$$A \cdot a \cdot N,$$

we have evidently $\frac{Aav}{l}$ = expenditure of steam.

Therefore equation (2) becomes

$$\frac{Aav}{l} = mS \cdot \frac{P}{p},$$

$$\therefore p = \frac{mS \cdot Pl}{Aav}.$$

The whole work done per square inch both before and after expansion being

$$ap + ap \log . \frac{l}{a},$$

the pressure of the steam in the cylinder is evidently

$$A\left(ap + ap \cdot \log . \frac{l}{a}\right);$$

and if R be the resistance per square inch against the piston, the whole resistance will be expressed by

$$R \cdot A \cdot l,$$

hence, by the first principle we have

$$A\left(ap + ap \cdot \log . \frac{l}{a}\right) = R \cdot A \cdot l \quad \text{.................. (4)}$$

$$ap + ap \cdot \log . \frac{l}{a} = R \cdot l$$

$$ap\left(1 + \log . \frac{l}{a}\right) = R \cdot l$$

$$\text{but, } p = \frac{mS \cdot Pl}{Aav},$$

$$\therefore \frac{mSPl}{Av}\left(1 + \log . \frac{l}{a}\right) = R \cdot l;$$

therefore $$v = \frac{mSP}{AR}\left(1 + \log . \frac{l}{a}\right) \text{..................(5).}$$

This is the same expression as deduced by Pambour, at page 107, in a different manner, when the clearance is neglected.

When the clearance is taken into account, the work done per square inch before expansion is represented by ap; but that done after expansion becomes

$$p\,(a + c)\log.\left(\frac{l + c}{a + c}\right),$$

hence the first relation becomes

$$A\left\{p\,(a + c)\log.\left(\frac{l + c}{a + c}\right) + ap\right\} = R\,.\,A\,.\,l,$$

$$\text{or, } A\,.\,p\,(a + c)\left\{\frac{a}{a + c} + \log.\left(\frac{l + c}{a + c}\right)\right\} = R\,.\,A\,.\,l.$$

For the second relation we will have

$$\frac{A\,.\,v\,.\,(a + c)}{l} = mS\,.\,\frac{P}{p};$$

$$\therefore\ v = \frac{mSP}{AR}\,.\left\{\frac{a}{a + c} + \log.\left(\frac{l + c}{a + c}\right)\right\} \quad \ldots\ldots \ (6)$$

c being the whole clearance.

The relation between the pressure and temperature of steam is of great importance, and numerous experiments have been made to ascertain the pressure when the temperature is known, or to determine the temperature when the pressure is known.

The most extensive as well as the most delicate experiments ever undertaken, are those of Arago and Dulong, which were made at the expense of the French Government; these experiments range from 1 atmosphere up to 24, and the following formula has been given by those eminent philosophers, which will represent the temperature for pressures from 4 to 50 atmospheres, without any sensible error*.

* These formulæ differ a little from those given by Pambour; he

$$p = (\cdot 269704 + \cdot 0068031\, t)^5 \quad \text{.................} \quad (7)$$

$$t = 146\cdot 991 \sqrt[5]{p} - 39\cdot 644$$

where p is the pressure in pounds per square inch, and t the temperature in degrees of Fahrenheit.

Tredgold's formula, as modified by Mellet, for pressures from 1 to 4 atmospheres,

$$p = \left(\frac{103 + t}{201\cdot 18}\right)^6 \quad \text{..............................} \quad (8)$$

$$t = 201\cdot 18 \sqrt[6]{p} - 103.$$

The formula by Pambour for the same range, is

$$p = \left(\frac{98\cdot 806 + t}{198\cdot 562}\right)^6 \quad \text{............................} \quad (9)$$

$$t = 198\cdot 562 \sqrt[6]{p} - 98\cdot 806.$$

Another useful property of elastic fluids has been discovered by the celebrated Gay Lussac; viz. that if the temperature of a given weight of any elastic fluid be made to vary, its tension being the same, it will receive augmentations of volume exactly proportional to the augmentations of temperature, and for every increase of one degree of temperature (Fahrenheit) will be produced an increase of ·00202 of the volume occupied by the fluid at the temperature of 32°.

If v be the volume of any given weight of elastic fluid under any pressure, and at the temperature of 32° of Fahrenheit, the volume it will occupy under the same pressure and at the temperature t, of Fahrenheit, will be

$$v' = v + v \times \cdot 00202\,(t - 32) \quad \text{................} \quad (1)$$

has made some slight mistake in converting the French measures into English.

We omit *Southern's* formulæ, as they are of no practical importance; also *Biot's*, though they apply with great accuracy; for the labour of calculation is so very complicated, that it makes them utterly unfit for a work of this kind.

Also, for the volume v'' and temperature t', we have

$$v'' = v + v \times \cdot 00202\,(t' - 32) \;\ldots\ldots\ldots\ldots\ldots\ldots \;(2)$$

$$\therefore \frac{v'}{v''} = \frac{1 + \cdot 00202\,(t - 32)}{1 + \cdot 00202\,(t' - 32)}.$$

This will also be true, if, instead of the ratio of the absolute volumes v and v'', we put the ratio of the relative volumes u and u'; suppose

$$\frac{u}{u'} = \frac{1 + \cdot 00202\,(t - 32)}{1 + \cdot 00202\,(t' - 32)}.$$

Now, neither the law of Mariotte nor that of Gay Lussac can apply alone to the changes which take place in the steam whilst in contact with the fluid from which it was evaporated, but from the two a third relation may be deduced, by which we may determine the variations of the volume of steam when there is a change in both the temperature and pressure at the same time.

If it were required to find the volume occupied by a given weight of steam which passes from the pressure p' and temperature t' to the pressure p and temperature t, we may suppose that the steam passes first from the pressure p' to the pressure p, without changing its temperature; then, by Mariotte's law, we have

$$u'' = u' \frac{p'}{p}.$$

Now, let the steam be supposed to pass from the temperature t' to the temperature t, without changing the pressure; then, by Gay Lussac's law, we have

$$u = u'' \left\{\frac{1 + \cdot 00202\,(t - 32)}{1 + \cdot 00202\,(t' - 32)}\right\}$$

$$= u' \frac{p'}{p} \left\{\frac{1 + \cdot 00202\,(t - 32)}{1 + \cdot 00202\,(t' - 32)}\right\}$$

which gives the law of the relative volumes when both the temperature and pressure change together; if we substitute in the above equation the values of p and t, p' and

t', the pressures and temperatures which correspond to each other in steam in contact with the water, we shall have the relative volumes; it is known, that under the pressure of the atmosphere, or 14·706 lbs. per square inch, and at the temperature of 212° (Fahrenheit), the relative volume of steam in contact with the water is known to be 1700 times that of the water which has produced it.

$$u = 1700 \times \frac{14{\cdot}706}{p} \times \frac{1 + {\cdot}00202\,(t - 32)}{1 + {\cdot}00202 \times 180}.$$

By means of the above formula and those at page 102, we can find the relation between the volume of steam and the pressure.

From 1 to 4 atmospheres it is best to use Tredgold's formula; and from 4 and upwards, that of Arago and Dulong.

In order to show the great accuracy of these formulæ, we shall work out some examples which may be compared with the Tables given at page 11, which have been found from actual experiments, and are given by Pambour.

To find the temperature when the pressures are repectively 20, 30, 40, 50, 60 lbs. per square inch.

By Tredgold's formula,

$$t = 201{\cdot}18 \sqrt[6]{p} - 103.$$

$$\text{Log}\,.\,t = (\log.\,201{\cdot}18 + \frac{1}{6}\log.\,20) - 103$$

$$= (2{\cdot}3035848 + 0{\cdot}2168383) - 103$$

$$= (2{\cdot}5204231) - 103$$

$$t = 331{\cdot}4 - 103 = 228^{\circ}{\cdot}4 \text{ Fahrenheit.}$$

In the same manner,

$$\text{Log}\,.\,t = (\log.\,201{\cdot}18 + \frac{1}{6}\log.\,30) - 103$$

$$= (2{\cdot}3035848 + 0{\cdot}2461869) - 103$$

$$= (2{\cdot}5497717) - 103$$

$$t = 354{\cdot}6 - 103 = 251^{\circ}{\cdot}6 \text{ Fahrenheit.}$$

$$\text{Log}\,.\,t = (\log\,.\,201{\cdot}18 + \frac{1}{6}\log\,.\,40) - 103$$

$$= (2{\cdot}3035848 + 0{\cdot}2670100) - 103$$

$$= (2{\cdot}5705948) - 103$$

$$t = 372 - 103 = 269^{\circ} \text{ Fahrenheit.}$$

$$\text{Log}\,.\,t = (\log\,.\,201{\cdot}18 + \frac{1}{6}\log\,.\,50) - 103$$

$$= (2{\cdot}3035848 + 0{\cdot}2831616) - 103$$

$$= (2{\cdot}5867464) - 103$$

$$t = 386{\cdot}1 - 103 = 283^{\circ}{\cdot}1 \text{ Fahrenheit.}$$

$$\text{Log}\,.\,t = (\log\,.\,201{\cdot}18 + \frac{1}{6}\,.\,\log\,.\,60) - 103$$

$$= (2{\cdot}3035848 + 0{\cdot}2963585) - 103$$

$$= (2{\cdot}5999433) - 103$$

$$t = 398 - 103 = 295^{\circ} \text{ Fahrenheit.}$$

To find the temperature when the pressures are respectively 75, 90, 135, 180, 240 lbs. per square inch.

Arago and Dulong's formula is

$$t = 146{\cdot}991\sqrt[5]{p} - 39{\cdot}644.$$

$$\text{Log}\,.\,t = (\log\,.\,146{\cdot}991 + \frac{1}{5}\log\,.\,75) - 39{\cdot}644$$

$$= (2{\cdot}1672908 + 0{\cdot}3750122) - 39{\cdot}644$$

$$= (2{\cdot}5423030) - 39{\cdot}644$$

$$t = 348{\cdot}580 - 39{\cdot}644 = 308^{\circ}{\cdot}936 \text{ Fahrenheit.}$$

$$\text{Log}\,.\,t = (\log.\,149{\cdot}991 + \frac{1}{5}\log.\,90) - 39{\cdot}644$$
$$= (2{\cdot}1672908 + 0{\cdot}3908485) - 39{\cdot}644$$
$$= (2{\cdot}5581393) - 39{\cdot}644$$
$$t = 361{\cdot}426 - 39{\cdot}644 = 321^{\circ}{\cdot}782 \text{ Fahrenheit.}$$

$$\text{Log}\,.\,t = (\log.\,146{\cdot}991 + \frac{1}{5}\log.\,135) - 39{\cdot}644$$
$$= (2{\cdot}1672908 + 0{\cdot}4260667) - 39{\cdot}644$$
$$= (2{\cdot}5933575) - 39{\cdot}644$$
$$t = 392{\cdot}064 - 39{\cdot}644 = 352^{\circ}{\cdot}420 \text{ Fahrenheit.}$$

$$\text{Log}\,.\,t = (\log.\,146{\cdot}991 + \frac{1}{5}\log.\,180) - 39{\cdot}644$$
$$= (2{\cdot}1672908 + 0{\cdot}4510545) - 39{\cdot}644$$
$$= (2{\cdot}6183453) - 39{\cdot}644$$
$$t = 415{\cdot}284 - 39{\cdot}644 = 375^{\circ}{\cdot}640 \text{ Fahrenheit.}$$

$$\text{Log}\,.\,t = (\log.\,146{\cdot}991 + \frac{1}{5}\log.\,240) - 39{\cdot}644$$
$$= (2{\cdot}1672908 + 0{\cdot}4760422) - 39{\cdot}644$$
$$= (2{\cdot}6433330) - 39{\cdot}644$$
$$t = 439{\cdot}879 - 39{\cdot}644 = 400^{\circ}{\cdot}235 \text{ Fahrenheit.}$$

To find the volume of the steam compared to the volume of the water that has produced it, when the pressure is 55 lbs.

Then by Tredgold's formula, the temperature will be 288°·4 Fahrenheit.

The formula for the volume is

$$u = 1700 \times \frac{14{\cdot}706}{p} \times \frac{1 + {\cdot}00202\,(t - 32)}{1 + {\cdot}00202 \times 180}$$

$$= 1700 \times \frac{14{\cdot}706}{55} \times \frac{1 + {\cdot}00202\,(288{\cdot}4 - 32)}{1 + {\cdot}00202 \times 180}$$

$$= \frac{25000{\cdot}2}{55} \times \frac{1{\cdot}517928}{1{\cdot}3636}$$

$$= \frac{37948{\cdot}5035856}{74{\cdot}998},$$

$$\therefore\ u = 505{\cdot}99.$$

Now, in all calculations for steam engines where expansion is used, it is clear that no one of the formulæ can be used so as to give correct results, for the pressure may vary from 4 or 5 atmospheres to considerably less than 1 atmosphere; besides, the calculation would be so complicated as to be of little practical use; this being the case, empirical methods have been used to give the pressure and volume in terms of each other.

Pambour* gives for condensing engines,

$$u = \frac{10000}{{\cdot}4227 + {\cdot}00258p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (a)$$

* The same weight of high-pressure steam will raise the temperature of a given weight of water in the same degree, whatever may be the pressure of the steam. This is commonly known as the law of Watt, and has been proved experimentally by M. Clement and others; and although the recent laborious experiments of M. V. Regnault show that it is not correct, it is, nevertheless, likely to maintain its position among practical men, on account of its simplicity and sufficient accuracy for ordinary purposes. The experiments of Pambour are said to confirm the law of Watt, inasmuch as steam, "under an absolute pressure, varying from 2·7 to 4·4 atmospheres, and escaping into the atmosphere under an actual pressure of from 1·4 to 1·03 atmosphere, presented at its issue exactly the same temperature as though it were saturated."

for non-condensing engines,

$$u = \frac{10000}{1{\cdot}421 + {\cdot}0023\,p} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (b)$$

where u is the relative volume, or the ratio of the volume of the steam to that occupied by the same weight of water, and p the pressure expressed in pounds per square foot.

Pole's formula is

$$P = \frac{24250}{V - 65} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (c)$$

$$\text{or,} \quad V = \frac{24250}{P} + 65;$$

here P is the pressure in lbs. per square inch, and V its relative volume compared with that of its constituent water.

Mr. Pole observes, that this formula may be adopted without considerable error throughout the range generally required in the Cornish engines, viz. from 65 lbs. to 5 lbs. per square inch.

Now, if we suppose a volume of water E to be converted into steam at a pressure p, and if M be the volume of steam which can be produced by it, then

$$u = \frac{M}{E} = \frac{1}{\alpha + \beta p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (d)$$

If, again, the same volume of water be converted into steam at a pressure p', and M' is the absolute volume of steam at that time, we shall have

$$\frac{M'}{E} = \frac{1}{\alpha + \beta p'};$$

therefore, between the absolute volumes of steam which correspond to the same weight of water, we shall obtain, by eliminating E, the relation

$$\frac{M}{M'} = \frac{\frac{\alpha}{\beta} + p'}{\frac{\alpha}{\beta} + p} \text{ or, } \left(\frac{\alpha}{\beta} + p\right) M = M' \left(\frac{\alpha}{\beta} + p'\right)$$

$$\therefore p = \frac{M'}{M}\left(\frac{\alpha}{\beta} + p'\right) - \frac{\alpha}{\beta}$$

α and β are the constants in Pambour's formula.

Now, by proceeding in the same manner with Pole's formula, we obtain the ratio of the volumes

$$\frac{M'}{M} = \frac{\frac{\alpha}{P'} + \beta}{\frac{\alpha}{P} + \beta},$$

where $\alpha = 24250$ and $\beta = 65$.

On the Work done by the Engine on the Piston, per minute.

Let E represent the number of cubic feet of water converted into steam per minute, A the area of the piston in square feet, l the whole length of stroke, a that part of the stroke before the steam is cut off, P the pressure in the boiler, p' the pressure in the cylinder before expansion, p the pressure at the xth foot of the stroke, c the total clearance, N the number of single strokes per minute, and U the work of steam on the piston per minute.

Now, each cubic foot of water converted into steam exists in the cylinder before expansion begins, under a pressure p'; it therefore occupies a relative space by equation (d) represented by

$$u = \frac{1}{\alpha + \beta p'}.$$

Now, the number of cubic feet of water which is evaporated in the boiler, and passes into the cylinder at every stroke of the engine in the form of steam, is represented

by $\frac{E}{N}$; therefore, the space occupied in cubic feet in the cylinder when the valve is closed and expansion begins, will be represented by

$$\frac{E}{N} \cdot \left(\frac{1}{\alpha + \beta p'}\right).$$

In like manner, the space occupied at the xth foot of the stroke when the pressure p' has become p, is represented by

$$\frac{E}{N} \cdot \left(\frac{1}{\alpha + \beta p}\right).$$

But the space in the cylinder filled by the steam before expansion begins, taken in cubic feet, is evidently represented by

$$A\,(a + c)$$

and, the space in the cylinder occupied by it at the xth foot of the stroke, by

$$A\,(x + c).$$

Hence we have the following relations:—

$$\frac{E}{N} \cdot \left(\frac{1}{\alpha + \beta p'}\right) = A\,(a + c),$$

$$\text{and, } \frac{E}{N} \cdot \left(\frac{1}{\alpha + \beta p}\right) = A\,(x + c);$$

whence, from the former equation,

$$N = \frac{E}{A\,(a + c)\,(\alpha + \beta p')} \quad \dots\dots\dots\dots\dots \; (e)$$

$$\text{and, } p' = \frac{E}{NA\beta\,(a + c)} - \frac{\alpha}{\beta}$$

from the latter equation,

$$N\,(\alpha + \beta p) = \frac{E}{A\,(x + c)},$$

$$\therefore \; p = \frac{E}{NA} \cdot \frac{1}{\beta(x+c)} - \frac{\alpha}{\beta} \quad \ldots\ldots\ldots\ldots\ldots (f).$$

But the work done per square foot upon the piston after expansion begins, is represented by

$$\int_a^l p \, . \, dx$$

substituting for p its value from equation (f),

$$\int_a^l p \, . \, dx = \frac{E}{NA\beta} \cdot \int_a^l \frac{dx}{x+c} - \int_a^l \frac{\alpha \, . \, dx}{\beta}$$

$$= \frac{E}{NA\beta} \, . \log . (x+c) - \frac{\alpha}{\beta} \, . \, x$$

$$\therefore \int_a^l p \, . \, dx = \frac{E}{NA\beta} \, . \log \left(\frac{l+c}{a+c}\right) - \frac{\alpha}{\beta}(l-a).$$

Also, since the pressure upon the piston before expansion begins is represented by p', and it describes a feet under this pressure, the work done on each square foot is represented by

$$p'a = \frac{E}{NA\beta} \cdot \left(\frac{a}{a+c}\right) - \frac{\alpha a}{\beta} \text{ from equation } (e);$$

therefore, the whole work done upon each square foot of the piston, is the sum of these two values, and

$$= \frac{E}{NA\beta} \cdot \left\{ \log . \left(\frac{l+c}{a+c}\right) + \frac{a}{a+c} \right\} - \frac{\alpha}{\beta} \, . \, l.$$

Multiplying by NA, we shall have the whole work done on the piston per minute, or

$$U = \frac{E}{\beta} \cdot \left\{ \log . \left(\frac{l+c}{a+c}\right) + \frac{a}{a+c} \right\} - \frac{\alpha NAl}{\beta} \ldots\ldots\ldots (g).$$

The engine having attained uniform motion, the work developed by the power per minute must be equal to that

developed by the resistance per minute. Hence, R being the whole resistance,

$$\frac{E}{\beta}\left\{\log.\left(\frac{l+c}{a+c}\right)+\frac{a}{a+c}\right\}-\frac{\alpha}{\beta}.NAl=NA.l.R$$

$$N.A.l.R+\frac{\alpha}{\beta}N.A.l=\frac{E}{\beta}.\left\{\log.\left(\frac{l+c}{a+c}\right)+\frac{a}{a+c}\right\}$$

$$\text{and, } N.A.l\left(R+\frac{\alpha}{\beta}\right)=\frac{E}{\beta}.\left\{\log.\left(\frac{l+c}{a+c}\right)+\frac{a}{a+c}\right\}.$$

Let v be the velocity of the piston per minute, then

$$v=N.l.$$

Substituting in the previous equation,

$$vA\left(R+\frac{\alpha}{\beta}\right)=\frac{E}{\beta}.\left\{\log.\left(\frac{l+c}{a+c}\right)+\frac{a}{a+c}\right\}$$

$$v=\frac{E}{A\beta\left(R+\frac{\alpha}{\beta}\right)}.\left\{\log.\left(\frac{l+c}{a+c}\right)+\frac{a}{a+c}\right\},$$

$$\text{or, } v=\frac{E}{A}.\frac{1}{\alpha+\beta.R}.\left\{\log\left(\frac{l+c}{a+c}\right)+\frac{a}{a+c}\right\}\ldots\ldots(h).$$

The quantity R in this equation, is the whole resistance acting on an unit of surface of the piston, which comprehends the resistance arising from the motion of the useful load, which call ρ, and from the friction which may be represented by $f+\delta\rho$, where f is the friction of the engine unloaded, and δ the quantity that that friction is augmented for each unit of useful load ρ; also, let k be the resistance from imperfect condensation, then

$$R=\rho\,(1+\delta)+f+k.$$

Substituting this in equation (h), we have,

$$v = \frac{E}{A} \cdot \frac{1}{\alpha + \beta\{\varrho(1 + \delta) + f + h\}} \cdot \left\{\frac{a}{a + c} + \log\left(\frac{l + c}{a + c}\right)\right\} \dots (i).$$

If the engine does not work expansively, then

$$a = l$$

and, $v = \frac{E}{A} \cdot \frac{1}{\alpha + \beta\{\varrho(1 + \delta) + f + h\}} \cdot \left(\frac{l}{l + c}\right) \dots (k)$

for, $\log . \left(\frac{l + c}{a + c}\right)$ becomes $\log . \left(\frac{l + c}{l + c}\right) = \log . 1 = 0.$

Equation (i) is one of Pambour's fundamental equations: he determines by means of this equation, the load for a given velocity by finding ϱ from it; ϱ being the load or resistance for an unit of surface of the piston, the load on the whole piston is $A\varrho$.

$$A\varrho = \frac{E\left\{\log . \left(\frac{l + c}{a + c}\right) + \frac{a}{a + c}\right\}}{(1 + \delta)\beta v} - \frac{A}{1 + \delta}\left(\frac{\alpha}{\beta} + h + f\right) \dots (l).$$

and to find the evaporation of an engine to give motion to a load ϱ at the velocity v, we have from equation (i)

$$E = Av . \frac{\alpha + \beta\{(1 + \delta)\varrho + h + f\}}{\log\left(\frac{l + c}{a + c}\right) + \frac{a}{a + c}} \dots\dots\dots (m).$$

This gives the quantity of water which the boiler should be capable of evaporating per minute.

The useful effect which an engine can produce in an unit of time, at the velocity v, is evidently $A\varrho v$.

I

$$\therefore\ A\varrho v = \frac{E\left\{\log.\left(\frac{l+c}{a+c}\right) + \frac{a}{a+c}\right\}}{(1+\delta)\beta}$$

$$-\frac{Av}{1+\delta}\left(\frac{\alpha}{\beta} + h + f\right) \ldots (n).$$

To find the useful effect in terms of the load, multiply equation (i) by $A\rho$.

$$A\varrho v = \frac{E\varrho\left\{\log.\left(\frac{l+c}{a+c}\right) + \frac{a}{a+c}\right\}}{\alpha + \beta(1+\delta)\varrho + h + f} \ldots\ldots\ldots\ldots (o).$$

To find the horse-power of the engine, we only need divide by 33,000: we have before observed, that a horse is considered to be capable of raising that number of pounds one foot high in one minute.

Pambour takes $h = 4$ lbs. per square inch, and $f = \cdot 5$ lbs. and $h + f = 4\cdot 5$ lbs. per square inch, or, $4\cdot 5 \times 144 = 648$ lbs. per square foot, $c = \cdot 05\,l$, $1 + \delta = 1\cdot 14$ lbs.

From equation (e) and multiplying by l, we have

$$v = \frac{E}{(\alpha + \beta p')A} \cdot \frac{l}{a+c}.$$

Pambour puts for p' the pressure P in the boiler, and calls it the *velocity for the maximum of useful effect.*

$$v' = \frac{E}{(\alpha + \beta P)A} \cdot \frac{l}{a+c},$$

then by substitution we have

$$A\varrho' v' = \frac{E}{(1+\delta)\beta} \cdot \left\{\frac{a}{a+c} + \log.\left(\frac{l+c}{a+c}\right)\right.$$

$$\left. - \frac{l}{a+c} \cdot \frac{\alpha + \beta(h+f)}{\alpha + \beta P}\right\} \ldots (p);$$

but this can only be approximate, for the pressure of the steam in the cylinder can never equal that in the boiler.

By differentiating the equation for a, he finds at what part of the stroke the steam should be cut off so as to render this useful effect a maximum*, which gives

$$\frac{a}{l}=\frac{\frac{\alpha}{\beta}+h+f}{\frac{\alpha}{\beta}+P}=\frac{\frac{1}{\alpha+\beta P}}{\frac{1}{\alpha+\beta(h+f)}} \;\ldots\ldots\ldots\ldots (q).$$

or, in other words,

$$\frac{a}{l}=\frac{\text{volume at pressure } P}{\text{volume at pressure } P'}.$$

Pole's formula, which we have given at page 108, will be

$$\frac{a}{l}=\frac{\frac{\alpha}{P}+65}{\frac{\alpha}{h+f}+65},$$

α being equal to 24250.

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* $$A\rho' v'=\frac{E}{(1+\delta)\beta}\left\{\frac{a}{a+c}+\log.\left(\frac{l+c}{a+c}\right)-\frac{l}{a+c}\cdot\frac{a+\beta(h+f)}{a+\beta P}\right\}$$

must be a maximum.

Hence, throwing out constant quantities,

$$\frac{a}{a+c}+\log.\left(\frac{l+c}{a+c}\right)-\frac{l}{a+c}\cdot\frac{a+\beta(h+f)}{a+\beta P}=u$$

must also be a maximum.

Differentiating, we have

$$\frac{du}{da}=\frac{a+\beta(h+f)}{a+\beta P}\cdot\frac{l}{(a+c)^2}-\frac{a}{(a+c)^2}=0.$$

Therefore, $$\frac{a}{l}=\frac{a+\beta(h+f)}{a+\beta P}$$

$\frac{d^2u}{da^2}$ is evidently negative, indicating the maximum.

Then,

$$AH = R . \cos . \theta$$

$$HB = B'E = AB - AH = R - R . \cos . \theta.$$

In the same manner,

$$CK = r . \cos . \varphi$$

$$DK = C'F = CD - CK = r - r . \cos . \varphi.$$

By similar triangles,

$$\frac{B'P}{C'P} = \frac{B'E}{C'F}$$

$$\text{or, } \frac{x}{l - x} = \frac{R\,(1 - \cos\theta)}{r\,(1 - \cos\varphi)}$$

$$= \frac{R . \sin^2 \frac{\theta}{2}}{r . \sin^2 \frac{\varphi}{2}}$$

$$= \frac{r}{R} . \frac{R^2 . \sin^2 \frac{\theta}{2}}{r^2 . \sin^2 \frac{\varphi}{2}}$$

we may, without sensible error, assume

$$R . \sin . \frac{\theta}{2} = r . \sin \frac{\varphi}{2}.$$

$$\therefore \frac{x}{l - x} = \frac{r}{R},$$

$$\therefore x = \frac{lr}{R + r}.$$

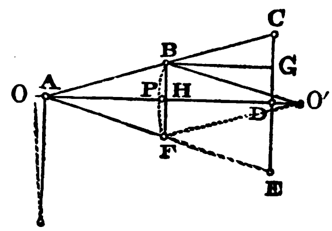
If the beams be equal, the point P will be in the middle of BE, but if unequal

$$AB : EO :: EP : BP$$

and, by composition,

$$AB + EO : AB :: EP + BP : EP\,,$$

extremity, half stroke and the lower extremity of the stroke respectively, and suppose BC to be any given distance; from D take $DP = BC$, and from E also set off $EF = BC$, then, through the three points B, P, E, describe a circle, the radius of which will be the length of radius rod required.



From this construction the following method of calculation is derived.

Draw BF parallel to CE, and BG to AD; then, since the triangles ACD and ABH are similar,

$$AC : CD :: AB : BH;$$

and, since AC, CD, and AB are given, BH is given, and BG being parallel to AD, $GD = BH$ and $CD - GD = CG$; hence, $\sqrt{BC^2 - CG^2} = BG = HD$ is given; but, $PD = BC$ by construction, hence, $PD - HD = PH$, the versed sine described by the radius rod is given; but $PH^2 + BH^2 = PB^2$; therefore, by Euclid, Cor. to Prop. 8, Book vi., $PB^2 \div 2PH =$ length of the radius rod is known.

There is yet another case that must be noticed, that is, when we are confined to have the end O' of the radius rod fixed in a certain position. This problem may thus be enunciated:—Given the length of the stroke, the length of the beam, and the distance of the centre O' from the vertical line the piston rod is required to move in, to find the length of the radius rod.

Let $AC = a$, $CD = b$, $O'D = c$, $AD = h$, and $BC = x$.

$$a : b :: x : CG \qquad \text{hence, } CG = \frac{bx}{a}$$

$$a : h :: x : BG \qquad \text{hence, } BG = \frac{hx}{a} = HD$$

$$O'D + DH = O'H = c + \frac{hx}{a}$$

$$CD - CG = GD = BH = b - \frac{bx}{a}$$

but, by 47 Prop. 1st Book of Euclid,

$$O'H^2 + BH^2 = O'B^2 = O'P^2;$$

$$\therefore \left(c + \frac{hx}{a}\right)^2 + \left(b - \frac{bx}{a}\right)^2 = (c + x)^2;$$

this equation solved, gives

$$x = \frac{ab^2}{2b^2 + 2ac - 2ch} = \frac{\frac{1}{2}a(a+h)}{a+h+c} = BC;$$

$$\therefore \text{ the length of radius rod } = c + \frac{\frac{1}{2}a(a+h)}{a+h+c}.$$

The calculation will be simpler if we neglect the length of the stroke.

$$\text{Since, } \frac{AB^2}{BC} = BO', \text{ we have } rx = (a - x)^2,$$

$$\text{but, } r = x + c;$$

$$\therefore x(x + c) = (a - x)^2;$$

$$\therefore x = \frac{a^2}{2a + c}, \text{ and, } x + c = c + \frac{a^2}{2a + c} = \frac{(a + c)^2}{2a + c},$$

the length of the radius rod.

If the radius rod be shorter than the parallel bar, we have

$$x(x - c) = (a - x)^2;$$

$$\therefore x = \frac{a^2}{2a - c}, \text{ and, } x - c = \frac{a^2}{2a - c} - c = \frac{(a - c)^2}{2a - c}$$

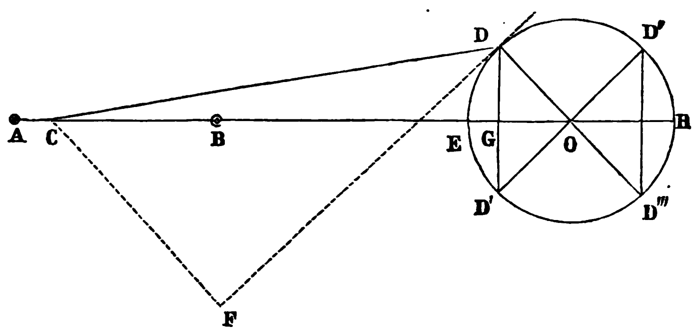
= the length of the radius rod.

From these equations are derived the rules given at page 23.

On the Crank.

In order to prove that there is no loss of power in the use of the crank in steam engines, as has already been stated at page 43, Mr. Woolhouse, in his *Appendix to Tredgold's Steam Engine*, gives the following dynamical investigation.

In the annexed figure let $CD = r$, be the length of the connecting rod, $OD = \rho$, the radius of the crank, α the



angle DOE described, v the velocity at C, v' the velocity at D, and the distance AC described $= x$. Then we shall have

$$DG = \rho . \sin . \alpha.$$

$$CG = \sqrt{r^2 - \rho^2 . \sin^2 . \alpha}.$$

$$OG = \rho . \cos . \alpha.$$

$$CO = \sqrt{r^2 - \rho^2 . \sin^2 \alpha} + \rho . \cos . \alpha.$$

$$AO = r + \rho.$$

and therefore, the value of AC or $AO - CO$, is

$$x = r + \rho - (\sqrt{r^2 - \rho^2 . \sin^2 . \alpha} + \rho . \cos . \alpha)$$

$$= \rho (1 - \cos . \alpha) + (r - \sqrt{r^2 - \rho^2 . \sin^2 . \alpha}) \;\ldots\ldots\; (1).$$

By differentiating this expression of the distance tra-

versed by C, and dividing by dt, the differential of the time, we obtain the velocity of the extremity C, viz.,

$$\frac{dx}{dt} = \frac{\varrho \, . \, d\alpha}{dt} . \sin . \alpha \left(\frac{\varrho \, . \cos . \alpha}{\sqrt{r^2 - \varrho^2 . \sin^2 . \alpha}} + 1 \right)$$

$\frac{dx}{dt}$ is the linear velocity of the point C,

$\frac{d\alpha}{dt}$ is the angular velocity of the point D,

and, consequently, $\frac{\varrho \, . \, d\alpha}{dt}$ is the linear velocity of the point D, therefore,

$$v = v' \sin . \alpha \left(\frac{\varrho \, . \cos \alpha}{\sqrt{r^2 - \varrho^2 . \sin^2 \alpha}} + 1 \right) \text{ .... } (2).$$

But if P denote the moving force acting at C, in the direction CB, P' the effective force at D, and c the angle DCO, the force P transferred in the direction of the connecting rod becomes

$$P . \sec . c;$$

and this resolved in the direction of the tangent to the circle, is

$$P . \sec . c . \sin . (c + \alpha),$$

and is equal to

$$P \, (\tan . c . \cos . \alpha + \sin . \alpha),$$

but,
$$\tan . c = \frac{\varrho \, . \sin . \alpha}{\sqrt{r^2 - \varrho^2 . \sin^2 . \alpha}};$$

$$\therefore P' = P . \sin . \alpha \left(\frac{\varrho \, . \cos . \alpha}{\sqrt{r^2 - \varrho^2 . \sin^2 . \alpha}} + 1 \right) \text{ ... } (3)$$

Equation (2) hence reduces to

$$v = v' \frac{P'}{P},$$

effect in the half-revolution as the connecting rod produces in the usual way.

$$\text{Let } x = AX, \text{ then}$$

$$2r \, . \, F = F \, . \, \pi x$$

$$\therefore x = \frac{2r \, . \, F}{F\pi} = \frac{2r}{\pi} = \frac{2r}{3{\cdot}1416} = {\cdot}637 \, . \, r.$$

On the Fly Wheel.

The illustrious Poncelet observes, that we should always consider what takes place in a complete revolution of the fly-wheel, and find the two positions of equilibrium, or those where the work of the power is equal to that of the resistance, for these two positions are where the velocity respectively becomes a maximum and a minimum. If we calculate the quantities of work expended by the power and absorbed by the resistance during the interval between these positions, and then put the double of that difference equal to the augmentation or diminution of the *vis viva* or living force of the fly-wheel, this relation, together with the condition that the greatest and least velocities shall not go beyond certain limits, leads us to the proper estimation of the dimensions of the fly-wheel.

Let F represent the force in the direction of the connecting rod, and Q the resistance that opposes the motion of the crank, and suppose that it acts tangentially to a wheel M, without inertia; also, let radius of the crank $= r$, the radius of the wheel $M = R$, R' the radius of the fly-wheel, P its weight, and V the velocity of the mean circumference; then,

$$\frac{P}{g} V^2,$$

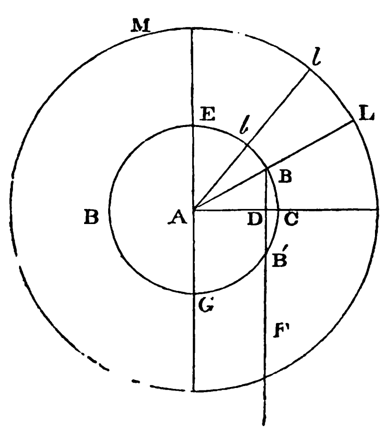
is the *vis viva* of the fly-wheel, or

$$\frac{P}{g} \, . \, R'v^2,$$

where v' is the angular velocity, which is proportional to the number of revolutions of the fly-wheel in a given time. For a given or the same number of revolutions, the *vis viva* of a fly-wheel increases proportionally to its weight and the square of its radius, or, in other words, for a radius *double*, *triple*, &c., the *vis viva* will become *four*, *nine*, &c., times as great, whence it follows we can augment the *vis viva* very considerably by increasing the radius.

The living force which is absorbed or accumulated in a fly-wheel is always equal to double the work done by the power diminished by that of the resistance; or it is sometimes expressed thus, the accumulated work is half the *vis viva*. Supposing all the circumstances of the work to remain the same, it is evident that the angular velocity will increase as much less as the weight and radius of the fly-wheel are made greater; we may, therefore, regulate the weight and size of the fly-wheel in such a manner, that the angular velocity may not pass a certain limit.

In the first place let us take the single-acting engine; suppose that it has attained the state of uniform motion, then, at the end of each revolution the power F shall be equal to the resistance Q, for, if the former were greater,



the velocity would increase from one revolution to another, and, consequently, the engine would not be properly regulated.

Now, the whole work of the power F during the half revolution or single stroke ECG is $F \,.\, EG = F \,.\, 2r$, and as the power does no work during the other half revolution, it is evident that $F \,.\, 2r$ represents the whole work of the power during a complete revolution. The work of the resistance Q, which acts during the whole revolution, and at right angles to the wheel M, whose radius is R, will be represented by $2\pi \,.\, RQ$; hence, by the first condition we have

$$F \,.\, 2r = 2\pi \,.\, R \,.\, Q;$$

$$\therefore \; Q = \frac{F \,.\, r}{\pi \,.\, R} \quad \text{......................... (1).}$$

Let us now find those positions of the crank where the velocity of the fly-wheel is greatest and least respectively.

We have before shown that the work of the power F at any point B is equal to $F \,.\, \frac{s}{r} \,.\, AD$ (page 125). Suppose that the centre of the crank pin is at E, and moves from left to right; at E the power is evidently nothing; although the resistance always acts, yet the crank continues to revolve, the engine being carried on by the accumulated work in the fly-wheel and the other moving parts; the work of the power continues to increase, but the velocity still decreases till the centre of the crank-pin arrives at some point B, between E and C, where the work of the power will become equal to that of the resistance, but whilst the centre of the crank-pin describes the same arc bB or s, the point of application of the resistance Q passes over an arc lL on the circumference of the wheel M, similar to bB, so that we have

$$lL : bB :: R : r$$

$$lL = \frac{R \,.\, bB}{r} = \frac{Rs}{r} \quad \text{...................... (2).}$$

The point B where the work of the power is equal to that of the resistance, is also that where the velocity ceases to decrease, and will be determined by the equality between the work of the power and that of the resistance:

$$F \cdot \frac{s}{r} \cdot AD = Q \cdot lL \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

Substitute for Q and lL their values from (1) and (2), and we have

$$F \cdot AD = \frac{Fr}{\pi R} \cdot R;$$

$$\therefore\ AD = \frac{r}{\pi} = \cdot 318 \cdot r \ldots\ldots\ldots\ldots\ldots (4.)$$

Therefore, if we measure from the centre A along the horizontal radius a distance $AD = \frac{318}{1000}$ of the radius, and from the point D raise a perpendicular, it will cut the circle in B, the point required.

After the crank-pin has passed the point B, the power becomes greater than the resistance; the motion will, therefore, be accelerated until it arrives at that point where we have again the work of the power equal to that of the resistance, when the velocity of the fly-wheel will have attained its maximum. This point is evidently below the horizontal radius, because, when the crank-pin is at C, the work of the power is the greatest possible, and must therefore decrease before it can become equal to the resistance. The position of the crank where its velocity is a maximum, is found by the same calculation to be as before, $AD = \cdot 318 \cdot r$, which shows that the points B and B', where the velocity of the fly-wheel is a minimum and a maximum, are upon the same chord BB', perpendicular to the horizontal radius, of which the distance from the centre is $\cdot 318 \cdot r$.

Let V be the maximum velocity of the mean circum ference of the fly-wheel per second, and v its minimum velocity, and P being its weight, then the accumulated work at the point of minimum velocity is

$$\tfrac{1}{2} \cdot \frac{P}{g} v^2,$$

and the work accumulated at the point of maximum velocity is

$$\tfrac{1}{2} \cdot \frac{P}{g} \cdot V^2,$$

the difference of these, or

$$\tfrac{1}{2} \cdot \frac{P}{g} (V^2 - v^2),$$

is the work accumulated between the points of minimum and maximum velocity; this work being evidently accumulated by the excess of the work of the power over that expended on the resistance between those points.

The work of the power F between those points of minimum and maximum velocity is

$$F \,.\, \text{chord } BB \text{ or } F \,.\, 2BD,$$

$$BD = \sqrt{AB^2 - AD^2} = \sqrt{r^2 - (\cdot 318r)^2} = \cdot 948r;$$

$$\therefore \; BB = 1\cdot 896r;$$

$$\therefore \; F \,.\, BB = 1\cdot 896 \,.\, r \,.\, F. \; \ldots\ldots\ldots\ldots \; (5.)$$

The work of the resistance Q during the same interval is evidently Q multiplied by the arc described by its point of application, whilst the crank-pin moves through the arc BB'; this work is, therefore, equal to

$$Q \,.\, \text{arc } BB' \,.\, \frac{R}{r}.$$

The arc whose sine is $\cdot 948 = 71° \, 26\tfrac{1}{2}'$;

$$\therefore \; \text{the arc } BB' = 142° \, 53';$$

$$180° : 142° \, 53' :: 3\cdot 1416r : x;$$

$$\therefore \; x = 2\cdot 4938r = \text{length of the arc } BB';$$

hence the work of the resistance becomes

$$Q \,.\, r \,.\, \frac{R}{r} \,.\, 2\cdot 4938 = Q \,.\, R \,.\, 2\cdot 4938.$$

Substituting in this expression the value of Q, given by equation (1), viz. $Q = \frac{Fr}{\pi R}$, we have

$$\frac{F.r}{\pi R} \times 2{\cdot}4938.R,$$

$$\text{or, } \frac{F.r \times 2{\cdot}4938}{\pi} = \frac{F.r \times 2{\cdot}4938}{3{\cdot}1416} = {\cdot}7938.F.r.$$

The excess of the work of the power over that of the resistance will therefore be

$$1{\cdot}896.F.r - {\cdot}7938.F.r = 1{\cdot}102.F.r.$$

But this excess is equal to the accumulated work;

$$\therefore \tfrac{1}{2}.\frac{P}{g}(V^2 - v^2) = 1{\cdot}102.F.r. \quad\text{............ (6.)}$$

If V' be the mean velocity of the mean circumference of the fly-wheel, its weight ought to be such that its velocity should neither increase nor decrease more than $\frac{1}{n}$-th; that is, neither the maximum nor minimum velocities should deviate more than $\frac{1}{n}$-th from the mean: it is clear that the maximum velocity will thus be $V' + \frac{V'}{n}$, and the minimum velocity $V' - \frac{V'}{n}$, or,

$$V = V' + \frac{V'}{n}$$

$$v = V' - \frac{V'}{n},$$

$$V^2 - v^2 = \left(V' + \frac{V'}{n}\right)^2 - \left(V' - \frac{V'}{n}\right)^2 = \frac{4\,V'^2}{n};$$

$$\therefore \tfrac{1}{2}.\frac{P}{g}.(V^2 - v^2) = \tfrac{1}{2}.\frac{P}{g}.\frac{4\,V'^2}{n} = 1{\cdot}102.F.r;$$

$$\text{or, } \frac{2P \,.\, V'^2}{gn} = 1 \cdot 102 \,.\, F \,.\, r;$$

$$\therefore \; PV'^2 = \cdot 551 \,.\, r \,.\, F \,.\, g \,.\, n. \quad \ldots\ldots\ldots\ldots\ldots \quad (7.)$$

From this equation we can determine the weight P of the fly-wheel; this can also be determined in terms of the horse-power, for if $H =$ horse-power, then $33000 \,.\, H =$ number of units of work done upon the piston per minute; and since $2r \,.\, F$ is the number of units of work done per stroke, or in one revolution, we have $2r \,.\, FN =$ number of units of work done per minute, N being the number of revolutions per minute;

$$\therefore \; 2r \,.\, F \,.\, N = 33000 \, H$$

$$r \,.\, F = \frac{33000 \, H}{2N} = \frac{16500 \,.\, H}{N}.$$

Substituting this value of $r \,.\, F$ in equation (7),

$$P \,.\, V^2 = \frac{\cdot 551 \times 16500 \, H}{N} \,.\, g \,.\, n.$$

$$P = \frac{9091 \cdot 5 \, H \,.\, g \,.\, n}{V'^2 N}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8.)$$

Now, if $R' =$ the mean radius of the fly-wheel, then

$$V' = \frac{2\pi \,.\, R'N}{60} = \text{mean velocity per second,}$$

$$\text{or, } V'^2 = \frac{4\pi^2 \,.\, R'^2 N^2}{3600},$$

substituting this value in equation (8)

$$P = \frac{9091 \cdot 5 \,.\, H \,.\, g \,.\, n}{N} \times \frac{3600}{4\pi^2 \,.\, R'^2 N^2};$$

but, $g = 32\frac{1}{6}$ feet per second,

$$\therefore \; P = \frac{9091 \cdot 5 \times 3600 \times 32\frac{1}{6} \,.\, H \,.\, n}{4 \times (3 \cdot 1416)^2 R'^2 N^3}$$

$$= \frac{26566181 \cdot 18 \,.\, H \,.\, n}{R'^2 N^3} \text{ in lbs.} \quad \ldots\ldots\ldots\ldots\ldots \quad (9.)$$

or, $P = \frac{11860 . H . n}{R'^2 N^3}$ in tons. (Rule 7, page 30.) ... (10.)

If $A =$ area of a section of the rim, w the weight of each cubic foot of the fly-wheel, then, by the property of Guldinus,

$$2\pi R'A = \text{solid content of the rim};$$

$$\therefore\ P = 2\pi R' . A . w. \text{ ....} (11.)$$

Substituting this value for P in equation (9),

$$2\pi R' . A . w . = \frac{26566181 \cdot 18 . H . n}{R'^2 N^3};$$

and, as a cubic foot of cast iron weighs 450 lbs., we have

$$w = 450;$$

and, $R'^3 = \frac{26566181 \cdot 18 . H . n}{2 \times 3 \cdot 1416 \times 450 . A . N^3} = \frac{9395 \cdot 8 . H . n}{A . N^3};$

$\therefore\ R' = \frac{21 \cdot 1}{N} . \sqrt[3]{\frac{H . n}{A}}$. (Rule 8, page 31.) ... (12.)

Also, the area of the section is

$A = \frac{9395 \cdot 8 . H . n}{R'^3 N^3}$. (Rule 9, page 31.) ...... (13.)

If N represent the number of single strokes, then $\frac{N}{2}$ will be the number of revolutions; and from equation (10),

$$P = \frac{11860 . H . n}{R'^2 \left(\frac{N}{2}\right)^3},$$

or, $P = \frac{94880 . H . n}{R'^2 N^3}$ in tons. (Rule 10, page 32.)...(14.)

By equation (9),

$$2\pi R' . A . w = \frac{26566181 \cdot 18 . H . n}{R'^2 \left(\frac{N}{2}\right)^3},$$

$$R'^3 = \frac{75166 \, . \, H \, . \, n}{A N^3};$$

$$\therefore \; R' = \frac{42 \cdot 2}{N} \sqrt[3]{\frac{H \, . \, n}{A}} \text{ (Rule 11, page 32.).. (15.)}$$

$$\text{Also, } A = \frac{75166 \, . \, H \, . \, n}{R'^3 N^3} \text{ (Rule 12, page 33.) ... (16.)}$$

In the double-acting engine, the fly-wheel is calculated in a similar manner. The work done by the power during one revolution is $4r \, . \, F$; hence, in this case, we have

$$4r \, . \, F = 2\pi R \, . \, Q;$$

$$\therefore \; Q = \frac{4r \, . \, F}{2\pi R} = \frac{2r \, . \, F}{\pi R} \text{............. (17.)}$$

And by reasoning in the same manner as before, we find

$$AD = \cdot 6366 \, . \, r;$$

$$\therefore \; BD = \sqrt{AB^2 - AD^2} = \sqrt{r^2 - (\cdot 6366 \, . \, r)^2}$$

$$= \sqrt{\cdot 59474044 \, r^2} = \cdot 77119 \, r.$$

The arc whose sine is $\cdot 77119$, is rather more than $50° \; 27'$;

$$\therefore \text{ arc } BB' = 100° \; 54',$$

$$180° : 100° \; 54' :: 3 \cdot 1416 : x$$

$$x = 1 \cdot 761 \, r = \text{the length of the arc } BB'.$$

Now, the work of the resistance is as before, represented by

$$Q \, . \, \text{arc} \, . \, BB' \, . \, \frac{R}{r}$$

$$\text{or, } Q \, . \, r \, . \, \frac{R}{r} \times 1 \cdot 761 = Q \times 1 \cdot 761 \, . \, R$$

$$\text{but, } Q = \frac{2r \, . \, F}{\pi R} \text{ by equation (17);}$$

$$\therefore\ Q \times 1{\cdot}761 . R = \frac{2r . F}{\pi R} \times 1{\cdot}761 . R$$

$$= 1{\cdot}761 . r . F \times \frac{2}{\pi} = 1{\cdot}761 \times {\cdot}6366 . r . F$$

$$= 1{\cdot}1211 . r . F.$$

The work of the power in the same interval is F multiplied by the chord

$$BB' = F \times {\cdot}77119 . r \times 2 = 1{\cdot}5424 . r . F.$$

Hence the accumulated work in this interval is

$$\tfrac{1}{2} . \frac{P}{g}(V^2 - v^2) = (1{\cdot}5424 - 1{\cdot}1211)\, r . F = {\cdot}4213 . r . F.$$

$$\text{or, } \frac{2P . V'^2}{gn} = {\cdot}4213 . r . F. \quad \ldots\ldots\ldots\ldots\ldots\ldots (18.)$$

Now, $2r . F =$ the work done in a single stroke, hence, $4r . F =$ work done in one revolution of the fly-wheel;

$\therefore\ 4r . F . N =$ work done per minute,

but, $33000 H =$ work done per minute;

$$4r . F . N = 33000 H;$$

$$\therefore\ r . F = \frac{33000 H}{4N} = \frac{8250 . H}{N} \ldots\ldots\ldots\ldots (19.)$$

Substituting this value of $r . F$, in equation (18),

$$\frac{2PV'^2}{gn} = {\cdot}4213 . r . F = {\cdot}4213 \times \frac{8250 . H}{N};$$

$$\therefore\ P = \frac{{\cdot}4213 \times 8250 H . g . n}{2NV'^2} = \frac{1737{\cdot}8625 H . g . n}{NV'^2},$$

but, $$V'^2 = \frac{4\pi^2 R'^2 N^2}{3600};$$

$$\therefore\ P = \frac{1737{\cdot}8625 . H . g . n}{N} \times \frac{3600}{4\pi^2 R'^2 N^2}.$$

[illegible] for [illegible] its value [illegible] feet per second,

$$P = \frac{[illegible] \cdot H \cdot n}{[illegible]}$$

$$= \frac{[illegible] \cdot H \cdot n}{[illegible]} \text{ in lbs.} \quad \ldots\ldots\ldots\ldots\ldots\ldots (20.)$$

$$= \frac{[illegible] \cdot H \cdot n}{[illegible]} \text{ in tons. (Rule [illegible], page 27.)} \quad \ldots (21.)$$

Substituting the value from equation [illegible] in equation [illegible] we have

$$[illegible] \cdot x = \frac{[illegible] \cdot H \cdot n}{[illegible]}$$

$$D^2 = \frac{[illegible] \cdot H \cdot n}{[illegible]} = \frac{[illegible] \cdot H \cdot n}{[illegible] \cdot A \cdot w}$$

and $r =$ [illegible] lbs., the weight of a cubic foot of iron.

$$\therefore D^2 = \frac{[illegible] \cdot H \cdot n}{[illegible] \cdot A} = \frac{[illegible] \cdot H \cdot n}{V^2 \cdot A},$$

$$\text{or } D = \sqrt{\frac{[illegible] \cdot H \cdot n}{V^2 \cdot A}}$$

$$D = \frac{[illegible]}{V}\sqrt{\frac{H \cdot n}{A}} \quad \text{(Rule 1, page 28.)} \ldots (22.)$$

Also, the area of the section is

$$A = \frac{[illegible] \cdot H \cdot n}{[illegible]} \quad \text{(Rule 3, page 28.)} \quad (23.)$$

If we had put N for the number of single strokes, and therefore $\frac{N}{2}$ for the number of revolutions, the above would have been

$$P = \frac{2275 \cdot H \cdot n}{R^2 \left(\frac{N}{2}\right)^2} = \frac{9100 \cdot H \cdot n}{R^2 N^2} \text{ in tons.} \ldots (24.)$$

(Rule 4, page 29.)

$$R' = \frac{12\cdot17}{\frac{N}{2}} \sqrt[3]{\frac{H \, . \, n}{A}} = \frac{24\cdot34}{N} \sqrt[3]{\frac{H \, . \, n}{A}} \ldots (25.)$$

(Rule 5, page 29.)

Also, the area of the section of the rim,

$$A = \frac{14423 \, . \, H \, . \, n}{R'^3 N^3}. \quad \text{(Rule 6, page 30.)} \quad (26.)$$

Equation (21) gives the weight in tons of a fly-wheel of a given mean radius R', so that, being applied to an engine whose horse-power is H, making a given number N, of revolutions per minute, it shall cause the angular velocity of the wheel not to vary more than $\frac{1}{n}$th from the mean angular velocity. The same may be said of equation (24), taking $\frac{N}{2}$ for the revolutions per minute, instead of N.

Similar observations apply to R' and A. It is evident that the weight of the wheel varies inversely as the cube of the number of strokes per minute, so that an engine making twice as many strokes as another of equal horse-power, would receive an equal steadiness of motion from a fly-wheel of one-eighth the weight, the radius being the same.

On the Friction of a Fly-wheel.

Let ϕ = limiting angle of resistance, then $\tan \phi$ is the coefficient of friction, and $P . \tan \phi$ is the resistance from friction at the surface of the shaft; but in the time that the fly-wheel makes one revolution, this resistance acts through the whole circumference of the shaft $= 2\pi r$, if r be the radius of the shaft; hence, the work expended on friction is

$$2\pi r \, . \, P \tan \phi;$$

and if N be the number of single strokes per minute, and, therefore, $\frac{N}{2}=$ number of revolutions of the fly-wheel per minute, the work consumed by friction per minute is

$$NP \,.\, \pi r \,.\, \tan.\, \varphi$$

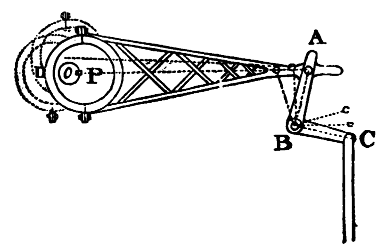
and in horses' power

$$\frac{NP \,.\, \pi r \,.\, \tan.\, \varphi}{33000}.$$

On the Eccentric Wheel.

Let S represent the space the end A is moved through by the eccentric wheel, and s the space the slide moves.

Then $\qquad AB \times \quad = BC \times S.$



Now, if this equation be solved for AB, BC, S, and s respectively, we have the following values:—

$$AB = \frac{BC \times S}{s}. \quad \text{(Rule 1, page 32.)}$$

$$BC = \frac{AB \times s}{S}. \quad \text{(Rule 2, page 33.)}$$

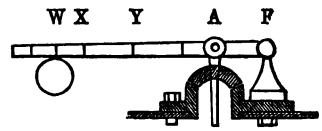
$$S = \frac{AB \times s}{BC}. \quad \text{(Rule 3, page 33.)}$$

$$s = \frac{BC \times S}{AB}. \quad \text{(Rule 4, page 33.)}$$

On the Safety Valve Lever.

The calculations for the safety valve levers, when the weight of the lever is taken into consideration, can be done in a shorter manner than by the rules given on the subject at pages 39 and 40, by employing the principles of the equality of moments.

Suppose Y to be the middle or centre of gravity of the lever, and if P be the pressure of steam against the valve, then



$$W \times WF + Y \times YF + A \times AF = P \times AF.$$

Taking the example given at page 40, we shall have, by substituting the various values,

$$W \times 24 + 3 \times 12 + 3 \times 3 = 210 \times 3,$$

$$24\,W = 630 - 36 - 9 = 585;$$

$$\therefore\ W = \frac{585}{24} = 24{\cdot}375, \text{ the same as found before.}$$

To know how far from the fulcrum the weight must be placed to press 20 lbs. per square inch, let FX be the distance, then, W being known, we must find FX.

$$W \times FX + Y \times FY + \text{weight of valve} \times AF = 140 \times AF,$$

$$24{\cdot}375 \times FX + 12 \times 3 + 3 \times 3 = 140 \times 3,$$

$$24{\cdot}375 FX = 375$$

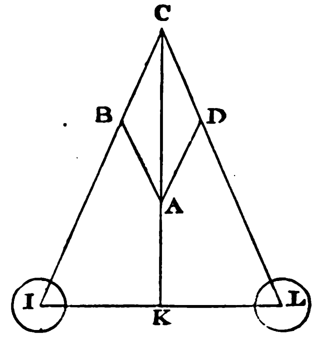
$$\therefore\ FX = \frac{375}{24{\cdot}375} = 15{\cdot}384, \text{ the same as found before.}$$

On the Governor.

We have said before, that in calculations for the governor there are two conditions to be fulfilled.

1st. To find the position of the point A of the slider, when the governor has the required angular velocity.

2nd. The centrifugal force of the balls should be capable of regulating the steam through the throttle valve, so as to bring back the engine, when the velocity becomes greater than that with which the engine moves when doing its usual work, to its proper speed.



To establish the first condition, we must consider that the balls are acted on by the centrifugal force, which acts in the direction KI, and the weight of the balls in the direction CK. Now, if CI represent the magnitude of the resultant, KI and CK will be proportional to the centrifugal force, and the weight P of the ball respectively.

Let $F =$ centrifugal force, then

$$\frac{F}{P} = \frac{KI}{CK}.$$

It is known that the centrifugal force of any body, which is small compared with its distance from the centre round which it revolves, is equal to the *vis viva*, divided by the radius of the circle, described by its centre of gravity*.

Now, if V' be the angular velocity of the weight P, when the governor has the required velocity, then

$$V' . KI$$

is the velocity of the centre of gravity of P, and the *vis viva* is equal to

$$\frac{P}{g} . V'^2 . KI^2$$

* See Hann's Theoretical and Practical Mechanics, page 156.

this, divided by the radius KI, gives

$$\frac{P}{g} . V'^2 . KI$$

for the centrifugal force of the ball P; substituting this value for F in the above equation, we have

$$\frac{\frac{P}{g} . V'^2 . KI}{P} = \frac{KI}{CK}$$

$$\therefore\ CK = \frac{g}{V'^2}.$$

The angular velocity V' is the velocity per second at an unit of distance from the centre; hence, if n be the number of revolutions per minute, $\frac{n}{60}$ is the number per second.

$$\therefore\ V' = \frac{2\pi n}{60} = \frac{\pi n}{30}$$

Substituting this for V',

$$CK = \frac{30^2 \times g}{\pi^2 n^2} = \frac{900 . g}{\pi^2 n^2} \text{ in feet} = \frac{900 \times 12 \times 32\frac{1}{6}}{\pi^2 . n^2}$$

$$= \frac{347400}{(3{\cdot}1416)^2 n^2} = \frac{35199}{n^2} \text{ in inches. (Rule, page 43.)}$$

If, as is usual, the governor makes thirty revolutions per minute, then $CK = 39{\cdot}1$, which is the length of the pendulum that vibrates seconds.

CK may also be determined in the following manner:—let t = time of one revolution when the governor is at its proper speed, then 2π will be the space passed over in this time at an unit of distance from the axis, and $\frac{2\pi}{t}$ is the space passed over at the same distance in an unit of time.

$$\therefore\ V' = \frac{2\pi}{t},$$

$$\text{and } CK = \frac{g}{V'^2} = \frac{g}{\frac{4\pi^2}{t^2}} = \frac{gt^2}{4\pi^2}$$

$$\therefore\ t = 2\pi\sqrt{\frac{CK}{g}}.$$

Having determined CK, it is easy to find the range of the balls or radius of the plane of revolution

$$KL = \sqrt{CL^2 - CK^2}. \quad \text{(Rule, page 43.)}$$

To fulfil the second condition, we may observe that the first only supposes that the balls are always at the same vertical distance CK, but if either the power overcomes the resistance, or the resistance the power, the balls will move further out in the former case, or move more in towards the axis in the other. Now, the throttle valve cannot be opened or shut without offering a certain resistance, which may be measured by finding what weight will produce this effect; let p be this weight, and as it acts vertically or in the same direction as the weight P, the above relation by the resolution of forces becomes

$$\frac{F}{P + p \cdot \frac{CD}{CI}} = \frac{KI}{CK}$$

but F, or the centrifugal force, has been shown to be

$$\frac{P}{g} \cdot V'^2 \cdot KI$$

$$\therefore\ \frac{\frac{P}{g} \cdot V'^2 \cdot KI}{P + p \cdot \frac{CD}{CI}} = \frac{KI}{CK}$$

$$\text{or, } \frac{P}{g} \cdot V'^2 \cdot CK = P + p \cdot \frac{CD}{CI}$$

In this equation, CK is known by the first condition; p, CD, and CI are also known; hence, we can determine the weight of each ball.

The balls of the governor will not fly out immediately, for, the velocity must augment before this takes place. Suppose the excess of this velocity above the mean velocity V', to be $\frac{1}{10} \cdot V'$, so that this velocity is

$$V'\left(1 + \frac{1}{10}\right) = \frac{11 \cdot V'}{10}$$

at the moment the valve opens; substitute this value for V', and we have

$$\frac{P}{g} \cdot V'^2 \cdot \left(\frac{11}{10}\right)^2 \cdot CK = P + p \cdot \frac{CD}{CI}$$

$$\frac{CK \cdot V'^2 \cdot \left(\frac{11}{10}\right)^2}{g} = 1 + \frac{p}{P} \cdot \frac{CD}{CI}$$

$$\therefore \frac{p}{P} = \frac{CI}{CD} \cdot \frac{\left\{CK \cdot V'^2 \cdot \left(\frac{11}{10}\right)^2 - g\right\}}{g}$$

but from the first condition,

$$CK = \frac{g}{V'^2}, \text{ or, } \frac{CK \cdot V'^2}{g} = 1;$$

$$\therefore \frac{p}{P} = \frac{CI}{CD} \cdot \left\{\left(\frac{11}{10}\right)^2 - 1\right\} = \frac{21}{100} \cdot \frac{CI}{CD}.$$

In the steam engine, CD is usually made equal to

$$\frac{2}{3} \cdot CI, \text{ or, } \frac{CI}{CD} = \frac{3}{2};$$

substituting this value for $\frac{CI}{CD}$, we have

$$\frac{p}{P} = \frac{3}{2} \times \frac{21}{100} = \frac{63}{200};$$

$$\therefore P = \frac{200 \cdot p}{63} = 3{\cdot}174 \cdot p, \text{ in lbs.}$$

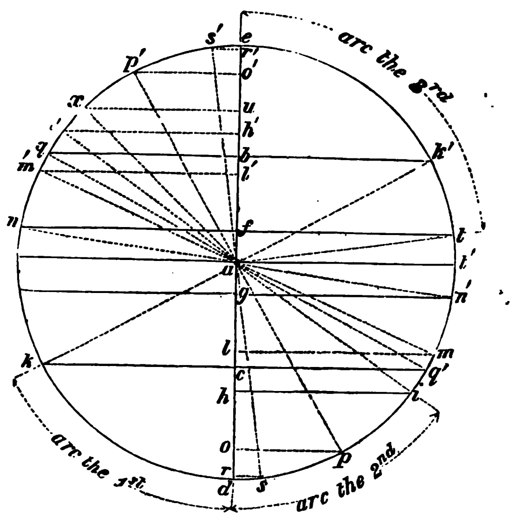
If $p = 10$ lbs., then $P = 31{\cdot}74$ lbs.

On the Lead of the Slide.

When a slide has lap and lead on both the steam and exhausting sides.

Let *a b* and *a c* represent the double lap on the steam side, *a f* and *a g* the same on the exhausting side, *b e* and *c d* the steam ports, and the line *e d* both the travel of the slide and the stroke of the piston; then, supposing *c h* to represent the lead of the slide, *a i* will be the position of the eccentric when that of the crank is *a e*, the piston being at the top of its downward stroke.

When the eccentric reaches the point *k*, the port *c d* will be fully closed and the piston will have descended to *l*, the arc *em* being equal to the arc *ik*. Again, when the eccentric



arrives at *n*, exhaustion commences from above the piston, which has descended to *o*, the arc *emp* being equal to the arc *ikn*. When the eccentric arrives at *q*, the port *be* begins to open for the admission of steam beneath the piston, which has descended to *r*, the arc *ems* being equal to the ark *ikq*. When the eccentric has reached the point

i', opposite to i, the port $b\,e$ will be open to the extent of the lead $b'\,h'$ equal to $c\,h$, and the piston will have completed its descent.

Steam continues to enter the port $b\,e$ during the ascent of the piston until the eccentric reaches the point k', when the port $b\,e$ will be reclosed, the direction of the slide's motion being downward and the piston having ascended to l'. Exhaustion ceases from above the piston when the eccentric reaches the point t, the piston being then at u. When the eccentric reaches the point n', opposite to n; exhaustion commences below the piston. Finally, when the eccentric reaches the point q', and the crank the point s', opposite to s, steam begins to enter the port $c\,d$, for the return stroke, at the commencement of which the port $c\,d$ will be open to the extent of the lead $c\,h$, the crank and eccentric occupying their original positions $a\,e$ and $a\,i$.

It is here shown that four distinct circumstances result from the use of a slide having lap on both sides of the port, with lead, during a single stroke of the piston. These are—

1. *The cutting off the steam for the purpose of expansion.*

2. *The cessation of the exhaustion on the exhaustion side.*

3. *The commencement of exhaustion on the steam side.*

4. *The readmission of steam for the return stroke.*

With regard to the first of the results, we found the steam port $c\,d$ closed when the crank and eccentric had described the equal arcs $e\,m$ and $i\,d\,k$. Now $c\,d$, the steam port, is the versed sine of $d\,k$; and $h\,d$, the steam port minus the lead, is the versed sine of $i\,d$.

Now, to reduce the versed sine of an arc from any given radius to radius unity, we must divide the versed sine to the arc by the radius of the arc, since the versed sines of arcs are as the radii of these arcs. Hence we have the first rule at page 53.

Exhaustion was shown to cease during the ascent of the

piston, when the eccentric had reached the point t, and the crank the point x, the crank having described the arc $d\,k\,x$ equal to $i\,d\,t$ described by the eccentric.

Now, $i'e$ is equal to arc the second, and $e\,t$ is equal to 90° minus $t\,t'$, or the arc whose versed sine is $e\,f$; and $e\,f$ is half the slide's travel minus the lap on the exhaust side. Hence the rule at the bottom of page 53, observing always that the versed sines are proportional to their radii. Exhaustion was shown to commence from above the piston, when the crank and eccentric had described the equal arcs $e\,k'\,p$ and $i\,d\,n$.

Now, $i\,d\,n$ is equal to 180° minus $n\,i'$, $n\,i'$ is equal to $n'\,i$, and $n'\,d$ is equal to arc the third. Hence the first rule at page 54. Steam was found to be readmitted for the return stroke when the piston had reached the point r in its descent, the crank and eccentric having described the equal arcs $e\,k'\,s$ and $i\,d\,q$. Now, $i\,d\,q$ is equal to 180° minus $q\,i'$, i' being diametrically opposed to i. And $q\,i'$ is equal to $i\,q'$, the difference between arcs the first and second. Hence the second rule at page 54.

On Paddle Wheels*.

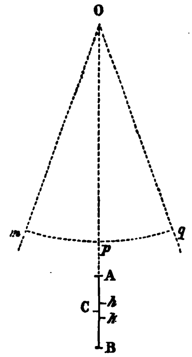
The difference of the curves during the lower part of the motion, amounts nearly to what is due to an arc described with a radius equal to the difference of the extreme radius of the wheel, and that of the circle of equal velocity with the ship.

I have considered from this cause, that the resistance, on any part of the float, varies nearly as the square of its distance from the rolling circle; and having at the same time taken into consideration the greater length of time of the action of the extremity than of the inner edge of the paddle, I find, from the examination of several experiments, that in the case of slight immersions the as-

* From Mr. Barlow's Paper in the Philosophical Transactions, vol. cxxiv. p. 315.

sumption of the resistance on any point varying as the cube of the distance from the rolling circle, and in deep immersions as the 2·5 power, will be a sufficiently near approximation for the present purpose.

Having thus assumed the ratio of resistance with respect to the radius, we readily find the position of the centre of pressure by the following equation.



Let $pA = r$ be the difference of the radius of the rolling circle and that of the wheel, n the power of the resistance in relation to the radius, $AB = b$ the depth of the paddle, $Ah = x$ any variable distance from its upper edge, $hk = dx$; $AC = y$ being the distance of the mean centre of pressure from the upper edge of the paddle: then

$$\int (r + x)^n . dx$$

will be the sum of all the resistances, and

$$(r + y)^n . b$$

the expression to which it is to be equal.

We have therefore, when

$$x = b$$

$$\frac{1}{n+1}(r + b)^{n+1} = (r + y)^n . b.$$

which, when $n = 3$, gives

$$y = \left\{\frac{(r + b)^4}{4b}\right\}^{\frac{1}{3}} - r. \text{ (Rule, page 80.)}$$

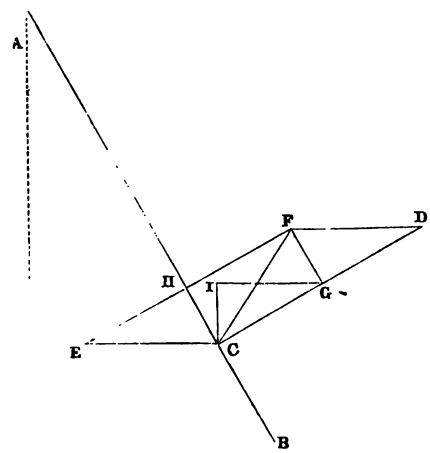
And, when $n = 2·5$,

$$y = \left\{\frac{2\,(r + b)^{\frac{7}{2}}}{7b}\right\}^{\frac{2}{5}} - r.$$

A striking difference is observable between the ratio of the resistance of the paddle in a vertical position to the power of the engine in the common wheels and in the new ones; the former being (in the Table) ·157 and ·193

with the large and small boats, and the latter ·546. This difference arises from the nature of their action. In the new wheels the vertical position is the most effective in propelling the vessel, and in the common wheels it is least so, as may be proved in the following manner:—

Let AB be the position of the paddle-rod of a vessel in motion, V being the velocity of the wheel, and v that of the ship, and φ the angle of inclination of the paddle-rod with a vertical line; let CD represent the velocity V at right angles to the paddle, and EC that of the vessel in a



horizontal direction. Then it is evident that CF, which is the resultant of these velocities, will represent the velocity and direction of motion of the paddle with respect to still water.

Resolve FC into the two velocities FG, CG, one at right angles to, and the other in the direction of the paddle, of which the latter is lost, while the former will represent the velocity with which the paddle meets the water in a direction at right angles to its face; then CG or $HF = EF - EH = V - v . \cos \varphi$. Consequently $(V - v . \cos . \varphi)^2$, will represent the whole resistance which the paddle opposes to the engine at any angle φ.

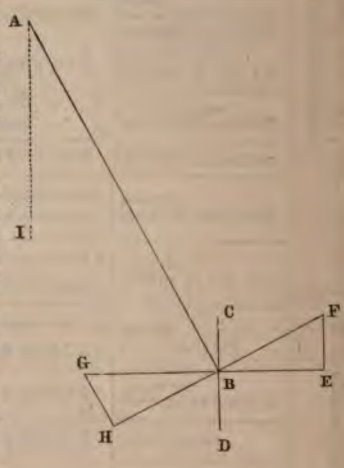
TABLE exhibiting the ratio of the Wheel and Vessel, the whole pressure upon the Paddle, and the results calculated from experiments made.

	Name of the Vessel.	Tonnage.	Horse-power.	Quantity of Coals, in chaldrons.	Diameter of the Wheel.	Length of the Paddle-boards.	Depth of the Paddle-boards.	Number of Paddles.	Dip of the extremity of the Paddle.	Strokes of the Engine, per minute.	Number of Strokes, per minute, for full power.	Speed of the Vessel, in English miles.	Area of the Paddle per horse-power.	Tons burden per horse-power.	Velocity of the Vessel, that of the Wheel being 1.	Diameter of the Rolling Circle.	Diameter to Centre of Pressure.	Pounds pressure upon the Vertical Paddle.	Proportion of the power of the Engine expended on the Vertical Paddles.	
					ft. in.	ft. in.	ft. in.		ft. in.							ft.	ft.	lbs.		
1	Messenger	730	200	60	19 4	10 0	2 0	16	..	20½	22	9·75	·200	3·65	·754	13·31	17·65	844	·154	
2	Messenger	730	200	130	19 4	10 0	2 0	16	..	18	22	8·00	·200	3·65	·730	12·80	17·53	886	·166	
3	Dee	710	200	30	19 4	10 0	2 0	16	1 6	23	22	10·61	·200	3·55	·732	12·91	18·00	1101	·207	
4	Rhadamanthus	820	220	46	20 4	9 0	2 0	16	..	20	22	10·39	·204	3·66	·791	14·54	18·36	720	·123	mean
5	Salamander....	820	220	210	20 4	9 0	2 6	16	5 6	15	22	8·15	·204	3·66	·828	15·21	18·36	268	·047	·157
6	Phœnix	820	220	12	20 4	9 0	2 6	16	2 6	26	22	11·70	·204	3·66	·840	15·60	18·57	468	·080	
7	Monarch	872	200	..	21 0	10 0	2 0	18	3 6	20½	22	10·72	·200	4·36	·748	14·62	19·55	1086	·220	
8	Monarch	..	..	..	..	..	..	..	..	20⅓	..	10·50	..	..	·746	14·60	..	1057	·217	
9	Monarch	..	..	..	..	..	..	..	..	21	..	11·02	..	..	·756	14·69	..	1061	·208	
10	Alban	294	100	14	13 0	9 0	1 6	14	..	27	30	8·84	·270	2·94	·377	9·15	11·77	354	·126	
11	Pluto..........	365	100	14	14 4	9 0	1 10	14	1 9	26½	30	10·15	·340	3·65	·823	10·71	13·01	308	·126	
12	Hermes........	730	140	130	17 6	9 0	2 0	..	..	18	24	6·30	·250	5·21	·626	9·80	15·66	1070	·277	
13	Meteor	296	100	8	13 0	9 0	1 6	..	1 6	32	30	9·00	·270	2·96	·671	7·87	11·70	1083	·370	·193
14	Firebrand	494	140	10	17 0	9 0	2 0	14	2 4	24	24	10·15	·250	3·51	·772	11·88	15·38	691	·178	
15	Firefly	550	140	152	17 6	9 0	2 0	14	3 4	20	24	8·30	·245	3·93	·733	11·60	15·81	684	·178	
16	Magnet........	360	140	6	16 0	10 0	1 6	..	1 8	29½	24	11·75	·214	2·57	·763	11·16	14·62	840	·149	
17	Carron	294	100	8	13 0	9 0	1 6	..	1 4	28	30	9·15	·270	2·94	·777	9·15	11·77	378	·137	
18	Medea	835	220	20	21 0*	4 10†	3 11	11	3 11	22½	22	11·33	·172	3·79	·627	13·79	22·03	3024	·666	
19	Flamer	494	120	13	13 0	5 9	2 9	9	3 11½	27	27½	10·90	·266	4·11	·683	11·30	16·55	1715	·625	
20	Flamer	..	..	112	..	..	..	..	5 6	24	27½	9·57	..	..	·674	11·16	16·55	1441	·526	·546
21	Firebrand......	494	120	12	14 6	4 6½	2 10	9	2 11½	28	27½	10·55	·212	4·11	·667	10·50	15·73	1508	·526	
22	Firebrand......	..	..	40	..	..	..	..	3 7	27	27½	10·10	..	..	·666	10·48	15·73	1404	·476	
23	Columbia......	360	100	80	14 0	3 11	3 0	9	4 10	24	30	8·50	·237	3·6	·654	9·91	15·15	1008	·454	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	

*Polygon. † Mean length.

In order to get an expression for the resistance in an horizontal direction, or that part of the power which is effective in propelling the vessel, CG must be resolved into the two resistances GI, CI, of which the former is $(V - v \cdot \cos\phi)^2 \cdot \cos\phi$; and it is to be shown that a mean resistance which would act uniformly through the arc ϕ, so as to be equal to this variable action, will exceed that of the mean action of the lower paddle; while in the new wheel the mean resistance is less than that of the lower paddle, and hence the great difference in the mean numbers in the foregoing Table.

In the new wheel, the paddle enters the water nearly in a vertical position; and, in order to simplify the investigation, I consider it to be truly vertical in every position, which is so near the truth in that part of the revolution when the action of the paddle takes place, that the results will be but slightly affected. Let CD be any position of a vertical paddle moving at a velocity V, in the direction FB of a tangent to the circumference. Then by resolving this velocity into two, one at right angles to, and one in the direction of the paddle, we find the velocity with which it meets the water at right angles to its face, to be $V \cdot \cos\phi$, ϕ being as before, the angle of inclination of the radius AB with a vertical.



The resistance opposed to the vertical paddle, when the ship is in motion with a velocity V, will therefore, be $(V \cdot \cos\phi - v)^2$; so that in the vertical paddle, when $V \cdot \cos\phi$ is equal to v, no resistance is opposed to the engine, and when it is less the paddle opposes a resistance

in a contrary direction; and it is sufficiently obvious that the resistance in every position in this case is less than when in its lowest position, while in the old wheel it is everywhere greater, at least within practical limits, which fully accounts for the difference in question.

It is observed above, that the resistance of the oblique paddle is always greater than in its vertical position, within the limits prescribed by practice. Let us examine what the actual limits are, by finding when with given velocities V and v, $(V - v . \cos\varphi)^2 . \cos\varphi$ is a maximum, or when

$$V . d\cos\varphi - 4 V . v . \cos\varphi . d\cos\varphi + 3 V^2 \cos v^2 . d\cos\varphi = 0;$$

whence,

$$\cos\varphi^2 - \frac{4 V . \cos\varphi}{3 v} = - \frac{V^2}{3 v^2},$$

and,

$$\cos\varphi = \frac{V}{3 v}.$$

It depends, therefore, on the relative velocities of the wheel and vessel.

When $V = 5$, $v = 4$, then $\varphi = 65^\circ{\cdot}33'$;

$V = 4$, $= 3$, $\varphi = 63^\circ{\cdot}37'$;

$V = 3$, $v = 2$, $\varphi = 60^\circ{\cdot}0'$.

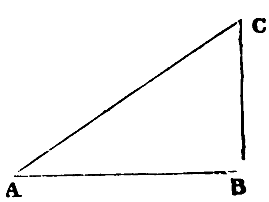
These results at once account for the ratio of the power of the engine to that of the resistance on the vertical paddle being greater in the old than in the new wheel. For it appears, contrary to the usual opinion, that not only the total resistance to the paddle increases as it deviates from the vertical, but that the effective horizontal force also increases up to all angles within the limits of the immersion of paddle wheels.

It should be stated, however, that although an increased propelling power is obtained from the vertical paddle upwards as far as these limits, it is not to be understood that so great an angle is practically advantageous; for the

vertical resistance becomes very great, and the shock on the engine by the paddles striking the water at so great an angle is tremendous.

ON THE SCREW.

We have before observed, that the screw is formed by wrapping an inclined plane round a cylinder.



In the inclined plane ABC, we have,

$$\frac{BC}{AB} = \tan . A,$$

$$\therefore \; BC = AB . \tan . A;$$

$$\text{but } AB = 2\pi r,$$

$$\therefore \; p = 2\pi r . \tan . A. \text{ (Rule, page 89.)}$$

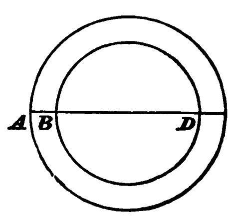
And since $2\pi r$ is a constant quantity, the pitch varies as $\tan . A$, the tangent of the angle of the screw.

ON WINDING ENGINES.

To find the diameter of a rope roll at first lift.

Let $x = BD$, the diameter of the roll at first lift; $t =$ the thickness of the rope; $b = AB$, the number of inches the radius of the roll is increased; and $p = \cdot 7854$.

Then, $px^2 =$ area of the small circle, and



$$p(x + 2b)^2 = px^2 + 4pbx + 4pb^2,$$

is the area of the large circle; their difference is $4pbx + 4pb^2$ $=$ the area of the annulus.

But the area of the annulus is equal to the area of the edge of the rope; that is, the

depth of the pit multiplied by the thickness of the rope. Let the depth of the pit in inches $= d$; then, $dt =$ area of the edge of the rope,

$$\therefore\ 4pbx + 4pb^2 = dt,$$

$$\text{or, } x = \frac{dt - 4pb^2}{4pb} = \frac{dt}{4bp} - b. \quad \text{(Rule, page 90.)}$$

To find where the corves will meet.

Let $r =$ radius of the roll; $R =$ the radius of the full roll; $n =$ the number of strokes or turns necessary to raise or lower the corves to meetings; $x =$ distance of meetings from the bottom of the pit; t and d, being as before, the thickness of the rope and the depth of the pit; $\pi =$ 3·1416.

Then the corves will be raised a height in n strokes or turns,

$$= \pi\{(2r + t) + (2r + 3t) + \text{\&c. to } n \text{ terms}\}$$

$$= \pi(2rn + n^2 t) = x. \quad \ldots\ldots\ldots\ldots \text{(1.)}$$

The corves descending will be lowered a depth

$$= \pi\{(2R - t) + (2r - 3t) + \text{\&c. to } n \text{ terms}\}$$

$$= \pi(2Rn - n^2 t) = d - x. \quad \ldots\ldots\ldots\ldots \text{(2.)}$$

$$\text{Also,} \quad \pi R^2 = \pi r^2 + dt. \quad \ldots\ldots\ldots\ldots \text{(3.)}$$

Adding (1) and (2) together,

$$\pi(2Rn + 2rn) = d$$

$$\therefore\ n = \frac{d}{2\pi(R + r)}.$$

Substituting this value in equation (1), we have

$$\pi\left\{\frac{rd}{\pi(R + r)} + \frac{d^2 t}{4\pi^2(R + r)^2}\right\} = x. \quad \ldots\ldots \text{(4.)}$$

But from equation (3),

$$R^2 = r^2 + \frac{dt}{\pi},$$

$$\text{or, } R = \sqrt{r^2 + \frac{dt}{\pi}};$$

this substituted in (4), gives

$$x = \frac{rd}{\left\{r + \sqrt{r^2 + \frac{dt}{\pi}}\right\}} + \frac{d^2 t}{4\pi\left\{r + \sqrt{r^2 + \frac{dt}{\pi}}\right\}}.$$

Hence the Rule at page 92.

On Steam.

Regnault observes that, "Mechanics have, for a long time, greatly desired a general investigation for the purpose of establishing these fundamental laws upon a series of direct experiments executed with the means of precision which physical sciences now present. I had for some time formed the determination of devoting myself to this work, and had several times tried some introductory experiments, which, however, served only to show me that precise results could only be obtained by means of a large apparatus, whose expense of construction far surpassed the very narrow means which we have at our disposition in our physical laboratories, and I should have been completely stopped in the execution of my projects, if the Minister of Public Works (upon the suggestion of M. Legrand, Under Secretary of State) had not, with a kindness which will be appreciated by all the friends of science, placed at my disposal the funds necessary for the execution of this long and laborious work*.

* As regards the varieties of steam, they may be designated as follows:—

Natural steam is that which is raised by solar heat.

Spheroidal steam is that which is raised by dropping water on hot metallic surfaces.

Surcharged steam is that which is raised by heating common steam when not in contact with water.

Common steam is that which is raised by ordinary heat.

"In order to show clearly what are the principal laws upon which the theory of steam engines rests, it appears to me necessary to explain, in a few words, the principles of this theory. All known systems of steam engines may be divided into four classes:

"1. Engines without expansion, and without condensation.

"2. Engines with expansion, and without condensation.

"3. Engines without expansion, but with condensation.

"4. Engines with both expansion and condensation.

"The first three classes may, in a theoretic point of view, be considered as particular cases of the fourth class, which presents the most complex case, the only one to which it is necessary for us to pay attention. We shall suppose an imaginary engine, which is not subjected to any external cause of cooling, nor to any loss of active force by friction, contractions of orifices, &c., &c. We shall suppose the boiler to be of very great capacity in comparison with the cylinder, so that the pressure of the steam may be considered as absolutely constant in the boiler during the motion of the machine; the heat of the furnace reproducing, constantly, the quantity of steam consumed by the machine.

"Let ω be the surface of the piston expressed, in square metres*.

"x, the space described by the piston from the instant of the arrival of the steam in the cylinder, with the tension which it has in the boiler, until the moment at which we are examining it.

"P, the constant pressure of the steam in the boiler,

* In the following translation we have preserved the French units of length, weight, and temperature. The metre is 39·371 inches; the kilogramme 2·205 lbs. av.

The degree of the Centigrade thermometer is 1·8 degrees Fahrenheit.

To reduce Centigrade to Fahrenheit degrees, multiply them by 9, divide the product by 5, and add 32 degrees.

expressed in kilogrammes and referred to a square metre of surface.

" T, the temperature of the steam.

" v, the capacity, in cubic metres, of the part of the cylinders described by the piston from its starting point to the height x.

" V, the total capacity of the cylinder.

" I. A first law, which it is important for us to know, is *the law which connects the elastic forces with the temperatures.*

" We will distinguish two periods during the stroke of the piston: during the first of these the cylinder communicates freely with the boiler; the total pressure of the steam upon the surface of the piston is $P\omega$.

" If the piston advances by a quantity dx, the element of work produced will be $P\omega dx = P dv$.

" The whole quantity of work produced during the first period, that is, from the beginning of the motion of the piston until the introduction of the steam is stopped (corresponding to a capacity V, described by the piston in the cylinder), is PV.

" During the second period, which is that of the expansion, no more steam comes from the boiler, but the steam contained in the cylinder continues to press upon the piston; as this rises, the steam occupies a larger space, its elastic force diminishes, and its temperature is lowered by the absorption of latent heat during its dilatation.

" Experiment has not decided what are the laws which govern these variations; but one of the following cases must happen:—

" *First case.*—The quantity of heat absorbed by a kilogramme of liquid water at 0° (32° Fahrenheit) in passing into vapour (which, for the sake of simplicity, we shall call *the total heat of the steam*), is the same, whatever may be the pressure, provided the vapour be at its maximum of density. If this law be exact, the steam will always remain in a state of saturation during the whole period of

the expansion; the pressures of the steam will vary in the inverse ratio of its volumes, and they will constantly present the relations to the temperatures, which connect the temperatures of saturated steam with its elastic forces.

"*Second case.*—The total heat of the steam increases in proportion as its elastic force is greater. As we suppose that the steam is not subjected to any external cooling influence, it is evident that, in proportion as the steam dilates into a larger space, it will require a smaller quantity of total heat to keep it in the state of vapour. Consequently, during the dilatation, there will be a disengagement of a certain quantity of latent heat, which will become sensible to the thermometer, and will raise the temperature of the steam above the point which corresponds to its saturation. The temperature of the steam will then be more slowly reduced than in the former case, the steam will be found overheated during the expansion, and the pressure of the steam upon the piston will diminish more slowly than it would according to the law of Mariotte.

"*Third case.*—The total heat of steam is less in proportion as its elastic force is greater. If this law were true, there would be a precipitation of liquid water during the expansion, the steam would remain constantly saturated, but the elastic force would decrease more rapidly than according to the law of Mariotte.

"In the absence of decisive experiments to show the accuracy of one of these three hypotheses, mechanicians have generally adopted the first, which is at the same time the most simple and the most precise. This hypothesis assimilates the expansion of steam to that of a permanent gas, dilating in a variable space, whose sides constantly restore to the gas the quantity of heat which is absorbed in the latent state during its expansion, so that its temperature remains invariable."

He gives the following formula, which is essentially the same as given at page 94.

Let v be the volume of steam, and p its pressure at any

given moment, dx the space described by the piston while the volume is increased by dv, the element of work produced will be $p\,\omega\,dx = p\,dv$; when the expansion begins, the volume is V and the pressure P, and as we admit the law of Mariotte, we have

$$p = \frac{PV}{v}, \qquad \therefore\ p\,dv = PV.\frac{dv}{v},$$

and the whole work produced while the volume of the steam passes from V to V', is

$$\int_V^{V'} PV\frac{dv}{v} = PV\log\frac{V'}{V} = PV\log\frac{P}{P'}.$$

This gives the work done by the expansion. The whole quantity of work done in one stroke is

$$PV\left(1 + \log\frac{P}{P'}\right).$$

We have here only taken into account the pressure on one side of the piston, the other side is constantly submitted to the pressure in the condenser; suppose this pressure to be constant during the whole stroke, and represented by f, the amount of resistance which it will have produced during that stroke will be

$$fV_1 = f\frac{VP}{P'},$$

so that the moving power will be

$$PV\left(1 + \log\frac{P}{P'} - \frac{f}{P'}\right).$$

If n represents the number of strokes per minute, the power developed during this unit of time will be

$$nPV\left(1 + \log\frac{P}{P'} - \frac{f}{P}\right).$$

The accuracy of the formula depends upon the accuracy of the hypothesis we have admitted, and it is necessary to determine it by direct experiments.

II. The quantities of heat which must be given to a kilogramme of water at 0°, to vaporize it at different pressures.

These quantities of heat are composed of two distinct parts—the heat necessary to raise the temperature of the liquid water from 0° to the point at which the change of state takes place, and the latent heat of vaporization. If we wish to distinguish these two parts of the total heat of steam, we must determine them by experiment.

III. *The capacity for heat of water at different temperatures.*

Finally, if the total heat of steam is not constant under all pressures, in order to calculate the effect of expansion, we must still learn

IV. *The specific heat of the vapour of water in different states of density, and at different temperatures.*

The theoretic power of the steam engine may be estimated by stating the amount of power which it is capable of giving for each kilogramme of steam consumed.

To do this, let ω be the weight of a cubic metre of steam under the pressure P, and temperature T; π the weight of steam consumed by the engine in one minute: we shall have $nV = \frac{\pi}{\omega}$; and, consequently, the power given by the engine from a kilogramme of steam will be

$$P\,\frac{\pi}{\omega}\left(1 + \log\frac{P}{P'} - \frac{f}{P'}\right).$$

But under all circumstances, in order to calculate the value of ω we must know

V. *The law according to which the density of saturated vapour of water varies under different pressures.*

VI. *The coefficient of dilatation of the vapour of water in its different states of density.*

Physical philosophers generally admit that the weight (ω) of a cubic metre of steam under the pressure P, and at the temperature T, may be calculated by applying to saturated steam the law of Mariotte and the law of the

uniform dilatation of the gases. Now, these laws are not even rigorously exact for the permanent gases, and it is to feared that they are completely false for saturated vapours.

Finally, the method most generally adopted to compare steam engines, consists in stating the work which they perform for each kilogramme of fuel consumed. To do this, we must know the weight (K) of steam under the pressure P, which a kilogramme of fuel can develope under the circumstances in which it is employed, and we have then for the work performed by a kilogramme of fuel,

$$P \, . \, K \, . \, \frac{\pi}{\omega}\left(1 + \log \frac{P}{P'} - \frac{f}{P'}\right).$$

The quantity K depends upon a variety of conditions, which we cannot now discuss, such as the quality of the fuel, the nature of the furnace, the arrangements of the boiler, &c., &c.

"To sum up, then, the theoretic calculation of steam engines requires the knowledge of the following laws and data:—

"I. The law which connects the temperatures and elastic forces of saturated steam.

"II. The quantities of heat which one kilogramme of liquid water at 0° absorbs, in being converted into saturated steam, under different pressures.

"III. The quantities of heat which one kilogramme of liquid water at 0° requires to elevate its temperature to that at which it assumes the state of steam, under different pressures.

"IV. The specific heat of aqueous vapour, in different states of density, and at different temperatures.

"V. The law according to which the density of saturated steam varies under different pressures.

"VI. The coefficients of dilatation of steam at different densities.

"Before commencing the search for these different laws, it was necessary to treat several preliminary questions so

as to fix with certainty the indispensable auxiliary data, and, above all, to define clearly the conditions which must be fulfilled by the thermometers, by means of which we measure the temperatures, in order that these instruments may be rigorously comparable.

"These preliminary researches obliged me to undertake, successively, long series of experiments, the necessity of which I was far from foreseeing when I undertook the work. I was, in fact, obliged to undertake the re-determination of a great number of data, which, for the most part, appeared to be fixed with complete certainty by the researches of my predecessors, and as to which physical philosophers entertain no doubts whatever."

On Stame.

Mr. Frost, of America, in a pamphlet "*On the Causes of the Explosions of Steam Boilers, and of some newly-discovered Properties of Heat*," has called attention to the greater economy of steam when heated between the boiler and cylinder, than when used in the ordinary way; steam heated in this manner he calls "stame," and considers that it is a different vapour, produced by different atomic proportions of heat and water from those which form ordinary steam.

It has been usual to treat steam as subject to the same laws as air, which expands about $\frac{1}{491}$ of its volume for each degree of Fahrenheit, but these experiments show an increase of seven volumes for an increase of about 438° of heat; if however, future experiments confirm these results, Gay Lussac's law for permanent gases is not adapted for heated steam. That law purports to commence at 32°, when the volume of water is unity, which at 212° becomes 1700 times that volume.

Mr. Frost states, "That they have obtained great advantage from improved boilers, with extensive surfaces giving time for the absorption of heat from the heated

smoke, which, being a non-conductor of heat, parts with its heat slowly; that they have obtained great advantages by husbanding the heat so obtained by incasing their boilers and cylinders with substances almost impervious to heat—are circumstances of considerable account: but still all those matters can constitute but a fraction of their great achievements, and their peculiar use of high steam in Watt's steam-jacket (an appendage which marine engineers superciliously overlook, though its employment is both indispensable and invaluable) will soon be found the greatest cause of the benefits the Cornish engineers have conferred on mankind.

"Engineers in general consider that the value of a steam-jacket consists alone of protecting the cylinder from the cooling influence of the air, or loss of heat by radiation, and have superseded it by incasing the cylinders of the best engines in felt and wood; now we shall presently show this proceeding is just as irrational and unintellectual as incasing a hungry starving animal with flannel, instead of supplying it with food."

Mr. Frost further states, with reference to some of his experiments, "How sensibly expansible (and therefore sensibly condensible) is steam by a minute addition of heat, just at the period the water assumes the elastic form, and, therefore, how greatly will the initial volume of steam be then affected by a minute addition or subtraction of heat. This being premised, it will become evident, that a steam-jacket is not only required to confine heat to the cylinder, but to continually furnish a supply of heat to the cylinder, so that the cylinder may in its turn furnish a continual supply of heat to the steam, while it is expanding within the cylinder, and thus greatly increase both its volume and tension, at little cost, which we have already shown it to do by the small quantity of heat required for the conversion of steam to stame.

"The main attempt of the Cornish engineers has been directed to obtain a greater duty by the employment of denser or hotter steam than Watt employed, and in order

(as they thought) to derive greater mechanical expansion, an[illegible]ere is little doubt they still congratulate themselves [illegible]ing, by that (inadequate) means alone, so greatly [illegible]ded; but the secret of the matter is, they have at [illegible]ne time derived another and greater benefit from the [illegible]al expansion or conversion of steam to stame, and [illegible] thus realized more than a double advantage, they [illegible]ight, and ought, to have suspected this occult cause [illegible]r success.

[illegible]at great philosopher, Boyle, justly observed, long [illegible]at if persons would disclose failures as well as suc- [illegible] science would progress far more rapidly. Follow- [illegible] advice, we will detail an error that will show both [illegible]ed and value of a steam-jacket, more than any [illegible]nt we can employ.

[illegible]n horizontal high-pressure steam-engine, having a [illegible]r 12 inches diameter, five feet stroke, the steam em- [illegible]d unexpansively, was altered (with the sole view of [illegible]g fuel) to an expansive engine, by exchanging the cylinder for one of more than 20 inches diameter, and therefore, of three-fold capacity. The workmanship was perfect, but the profit very small.

"We lately examined this engine (which is kept in constant work) by cementing some one-sided wood cups to different parts of the side of the cylinder, having filled those cups with fusible metal, which being in contact with the side of the cylinder, served to heat different thermometers to the same temperature as the different parts of the cylinder to which the fusible metal was applied.

"When the engine was using steam of 75 lbs. per inch above atmospheric pressure, and therefore = to 90 lbs. per inch, and temperature above 320°, while the steam was cut off at from $\frac{1}{4}$ to $\frac{1}{5}$ stroke, the temperature at the ends of the cylinder (which were alternately supplied with steam of 320° every third second of time) was found to be only 252°, and the temperature of the middle part of the sides of the cylinder was found to be only 212°; we also found on opening a cock, inserted into the head of the

M 2

cylinder, that the air rushed into that end of the cylinder while it was filled with the expanding steam therein, and at or before half the stroke of the engine had been accomplished.

"When it is considered that the density of the initial steam was such that the expanded steam ought to have had a greater tension than the atmosphere, and a greater temperature than 212° at the termination of the stroke, the want and value of a steam-jacket to this engine must be apparent to the dullest capacity, and yet no marine engine we have yet seen is furnished with such an appendage!

"Now we have shown why the Cornish engines are so vastly superior to all marine and manufacturing engines; we will next show how those latter engines may be caused as greatly to surpass the Cornish, which, being high pressure, and very high pressure engines, would be too dangerous for navigation, but more especially (be it particularly observed) that by the use of low steam converted to stame, both the greatest safety and greatest profit must be found connected; because the reasonable heating of either high or low steam (for conversion into the more profitable stame) must terminate at the same degree, it follows then that double profit will accrue from converting low steam to stame, because high steam of itself has been already so heated as to be incapable of more than half the profit to be derived from heating low steam.

"Having shown whence the Cornish engines have derived their vast superiority, that has so astonished the most eminent engineers, by the treble or quadruple duty, and which has appeared, as Mr. Palmer stated, both incredible, impossible, and incomprehensible; and having as plainly shown, that by heating steam apart from water, steam having but little capacity for heat, is greatly expanded by a very inconsiderable additional quantity of heat, becomes, a distinct and chemical compound of heat and water, 'stame;' having shown that by its production from low steam, it must become far more profitable than from high

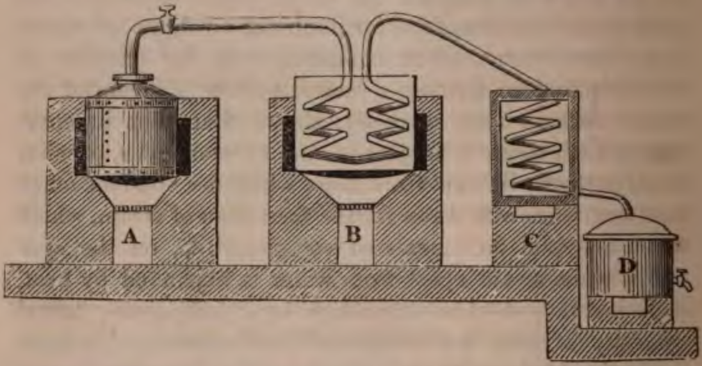
steam; having also shown, that from no other source can the present superior duty of the Cornish engines be derived—we shall take leave of this part of our subject by stating, that we have realized a duty of more than eighty millions from the use of stame in the condensing engine before described, whose cylinder was only six inches diameter, unprovided with any steam-jacket or other apparatus for expanding steam, by which means the duty would at least have been doubled, and in what a Cornish engineer would consider but a toy, which, nevertheless, shows the present general use of fuel for the production of motive force to be so inefficient, extravagant, and wasteful, as to be discreditable to the age."

In the Appendix, Mr. Frost observes, "Having seen the thermometrical degrees at which steam, apart from water, is expanded by heat into larger volumes, it becomes important to learn the actual quantity of heat required for each degree of expansion, and the apparatus represented by the following diagram will show, first, how small is the quantity of heat required for doubling a volume of steam apart from water, when compared with the quantity of heat required for forming a second volume of steam of the same tension; and, secondly, shows that heat in combining with steam is subject to, and controlled by, peculiar laws, perfectly distinct from those which are obtained when heat combines with water for the formation of steam, which requires equal increments of volume, while, on the contrary, when steam apart from water is expanded by heat, it is not only doubled in volume by a comparative trivial quantity of heat, but every additional increase of volume is obtained by a still smaller and rapidly-decreasing increment of heat, so that the greater the increase of volume, the smaller will be the quantity of heat required for that latest volume; and although this is so contrary to the general laws of heat, and therefore so adverse to common apprehension, the diagram and table will not only show it to be a chemical fact, but will furnish the easy means for any competent person to verify the fact, which must be

acknowledged to be of the first importance; for, were these facts understood, the present cost and weight of apparatus, and of fuel for the production of motive force, would both appear so extravagant, unscientific, and wasteful, as was the use of steam for motive force, before the days of Watt; yet, at that period, as at present, engineers conceived they fully understood the subject, '*oft attempted —never reached.*'

"Though it requires four times the force for double speed, it is evident, were the present enormous rate of fuel consumed in steamers judiciously applied, it would furnish abundant power for propelling them at much more than double speed, while the consumption of fuel for the voyage would, of course, be reduced to much less than one-half.

"*A*, furnace and steam boiler suspended over it. *B*, furnace and suspended steam heater for containing fluids boiling at stationary temperatures, and hollow worm con-



nected by a pipe and stop-cock with steam boiler *A*, and by a pipe with worm in *C*. *C*, a covered wood cistern, containing half a cubic foot of cold water and hollow worm therein.

"When a volume of steam from *A* was passed through the hollow worm in heater *B* (filled with water boiling at 212°), and into the hollow worm in *C*, until the condensed

water therefrom exactly filled a glass measure containing nearly twenty ounces of water, the heat separated from that definite volume of atmospheric steam heated the water in *C* 38°.

"When similar volumes of steam from *A* were passed through the worm in heater *B*, while the contained fluid was heated to the more elevated temperature in the Table, the excess of heat in each case above 38° showed the decreasing quantities of heat required for increasing the original volume of steam to the magnitude stated in the Table.

Temperature of boiler *A*.	Temperature of heater *B*.	Volumes of steam and stame produced at those heats shown in former experiments.	Temperature of water acquired in *C*, showing the different quantity of heat in different volumes of equal tension.	Comparative quantity of heat required for equal volumes of steam of equal tension.	The quantities of heat in 4, 5, 6, 7, 8 volumes being fractional, were incapable of exact definition, the 5 volumes requiring but 3·38 the quantity required for 1 volume of steam.
212°	212°	1	38°	38°	
212°	216°	2	42°	76°	
212°	228°	3	43°	114°	
212°	550°	8	46°	304°	

"This increasing force obtained from decreasing quantities of heat applied to steam apart from water, not only proves the prodigious economy of this means of obtaining motive force, but points out the physical cause of the superlative explosive force, attendant on greatly and suddenly heated elastic fluids.

"Many other and valuable advantages incidentally occurred during our experiments, which are omitted, because enough is given to stimulate the most torpid. We will, therefore, only add—

"The following advantages have been frequently verified by several of the most eminent engineers and learned and competent men of New York and other places, by a condensing engine and apparatus so constructed, when actuated alternately by common steam and by moderately-heated steam, so that the comparative quantities of heat

and of water actually employed for motive force in each separate experiment could be accurately measured as well as the power exerted by the engine.

"The general result showed, that more than six times the motive force was realized from equal quantities of heat and of water, when employed to actuate the engine with heated steam or stame, than was obtained from the use of natural steam—each being alike produced from the same constant fire and time, and the same engine; which engine, apparatus, scientific instruments described in this work, and testimonials of competent and respectable engineers, are open for inspection in Falton Avenue, near Gold Street, Brooklyn."

Dr. Haycraft, of Greenwich, had also made a number of experiments with stame, or, as he designates it, anhydrous steam, and freely criticised Frost's mode of conducting his experiments, as liable to error; at the same time, giving instances where he had himself been deceived in the results of experiments. He tried his plan on an engine having a nine-inch cylinder and three-feet stroke, which worked very economically; but he found that the pipes subjected to the heat gave way, and in the aggregate did not realise what he expected. He then employed a steam-jacket, and a vessel for separating the steam from any unevaporated water, and realised, in a large engine, 25 per cent. by the separation, and 46 per cent. economy where both steam-jacket and separation were used. These experiments were made before Mr. Wright, the Government Comptroller and Inspector of Steam Machinery. Although Dr. Haycraft differs with Frost on some details of the experiments, he yet fully admits the economy of stame to be very nearly as great as it is estimated by Frost.

The experiments of Mr. Frost attracted the attention of the Institute of Arts and Sciences at New York; a committee was formed, and after making a course of experiments, felt satisfied of the economy of stame if it could be

brought into operation, where the temperature of colder bodies would not interfere so as to abstract the heat before it could be profitably employed.

Spheroidal Steam.

We come now to what has been called spheroidal steam, which takes its name from the form water assumes on being dropped upon a red-hot metal plate, which plate having sides, or being so constructed as not to allow the drops to roll off, the drops then form themselves into spheroids. This was introduced into France about ten years ago, by M. Boutigny, and patented in this country by Beauregard, in 1848.

When the water is of a higher temperature than about 206°, the repellent power of the heat of the plate, and the sphere of the water, prevent their contact; and as may have been seen when a drop of water has fallen on a hot plate, it runs about, growing less and less until it disappears altogether, without being converted into steam.

The spheroidal generator was tried with a vessel of melted lead, heated to 540°, having a hemispherically indented platinum bottom plate, on which water was thrown from a pipe. The results were said to be more economical than with ordinary steam; but this may be doubted, as the real economy of ordinary steam enables it to compete with many ingenious productions of motive power.

Mr. Frost, in his celebrated work, explains the causes of boiler explosions in a way that was long ago explained in almost the same manner by Mr. Gurney:—see his evidence before the Committee of the House of Commons, in 1835. If any one will take the trouble of reading this evidence, we think that a striking resemblance will be found:—

"Will you state to the Committee your conception of the causes that led to those explosions?—My impression is, that in almost every case where a boiler has burst, there has been a want of sufficient water in it. From want of

water the sides have become heated to excess; when water is reduced too low in a boiler, the upper parts of the vessel become hot. If at this moment a call is made for the steam which is dissolved in that remaining, in its (the steam's) disengagement it throws up water in the form of froth, or bubbles, like the water which is expelled on taking the cork out of a bottle of soda-water. The water thus coming in contact with the heated sides of the vessel, instantly flashes into an unusual quantity of elastic steam; and the pressure of it explodes the boiler, notwithstanding a safety-valve of more than ordinary dimensions may be placed upon it at the time. I would illustrate my meaning by calling attention to the bursting of a gun-barrel: the whole area of the gun may be considered as the size of the opening of a safety-valve, and the charge of shot may be considered as the valve loaded, which perhaps, is about an ounce to a square inch. The quantity of elastic matter which is suddenly formed by the explosion of the gunpowder, though it may escape through the full diameter of the bore under a pressure of only an ounce to the inch, frequently bursts the strongest barrels. So also steam, which is similarly elastic matter, suddenly and largely formed in a boiler, notwithstanding the safety-valve may be almost equal to the area of the boiler itself, would sometimes burst it. In illustration, I would state a circumstance which once came under my notice, where an open boiler was burst by steam pressure. The accident happened in consequence of gelatinous matter dissolved in the water forming a film or blister, which blister prevented contact of the water at the under part of the boiler; it (this part of the boiler) consequently became red-hot; when the film or bubble burst, the sudden formation of steam burst out the bottom of the boiler, threw the water in all directions, and killed two men. A very rapid or sudden formation, therefore, of elastic matter will be sufficient to burst any boiler in ordinary working. If steam be kept within a boiler, and any portion of the boiler becomes intensely heated, instantly that the water

bubbles up (from its escapage), or comes in contact with the heated sides of the boiler, or mixed with the super-heated steam, it forms suddenly a quantity sufficient sometimes to rupture the strongest boiler. If the question was put with an idea that the decomposition of water into its elements, or that the separation of hydrogen gas, caused explosion of boilers, I beg to state that my observations do not lead me to conceive an accident to happen from such a cause. I have often burst vessels myself, experimentally, from the ebullition of steam, or rather the ebullition of water from the escape of steam, against heated sides, or up into the over-heated steam, or elastic matter."

The majority of accidents from explosion occur at the time of starting the engine. Now, although a boiler can bear the steady and constant pressure allotted to it without explosion, yet, when the steam is taken away to supply the cylinder, it has been found that the boiler has exploded. It seems to be something more than a paradox, to suppose thât by reducing the pressure, as we assuredly do by allowing the steam to escape either to the cylinder or to any other place, the boiler should explode in consequence of its weakness. If the boiler could sustain the constant pressure, say of 40 lbs. per square inch, is it likely that it will explode when that pressure is diminished by taking suddenly a considerable quantity of steam from it? This seems strange; but facts show that boilers generally explode under these apparently favourable circumstances. How is this to be accounted for?* Evidently from the

* One of the authors of this work was consulted professionally on the explosion of a boiler in the Island of Java. The boiler was erected for a sugar-mill; the regular pressure was 60 lbs. per square inch, and was proved or tested at three or four times that pressure. When the boiler was only pressed at 37 lbs. per square inch, the steam was drawn off from the boiler to the vacuum-pan; it immediately exploded, and was thrown from its seat horizontally. On observing how the boiler gave way, he was enabled to show in what direction it would really move, for it might move horizontally from left to right or from right to left, looking at the front of the boiler. It was a tubular one, and the collapse of the tube was from left

sudden formation of some matter producing a greater force, and a greater quantity also, than the safety-valve can relieve or the engine can withdraw in time to keep down that sudden generation of force which is sufficient to burst the boiler.

Mr. Dunn, of Worcester, delivered an excellent lecture on this subject, in the "Natural History Society's Room." The following are some of that gentleman's views:—

"The causes of steam-boiler explosions were usually described to be over-pressure, want of water in the boilers, or weakness in the materials of the boiler; but one thing had been quite overlooked, viz. over-temperature, which, and which alone, he believed, was the cause of steam-boiler explosions. They had all heard of explosions attributed to over-pressure, but nobody had ever heard of a steam-boiler bursting without a fire under it. No doubt the fact of a boiler being unsound might cause an explosion; but he would say, give him an unsound boiler, and for this reason—it generates steam slowly, it can only get on at a certain rate of progress, and at that it cannot explode. An over-rush of temperature was the cause of explosions, and that could only arise from a gradual accumulation of heat going on until the plates of the boiler got to a red heat. As far back as 1756, Pulteney, and later Klaproth, had discovered that there was a peculiar state of water in which, if placed upon a plate of metal at a red heat, it would run about, but not generate steam.

"Mr. Dunn illustrated this peculiar quality of water in a certain state of heat by a very convincing experiment. Having placed a shallow vessel over the flame of a spirit-lamp, until it became red-hot, boiling water was poured into it, and no steam followed; but the vessel having been removed from the spirit-lamp, a change of the temperature took place; it was lowered, of course, and then a little

to right, therefore the steam would issue in that direction. Hence, the boiler would be thrown in the opposite direction, viz. from right to left, on the same principle as Barker's mill, the flight of a rocket, or the kick of a gun.

cloud of steam rose from the water. It was a fact, that nine-tenths of all the accidents with steam-engines in the manufacturing districts happened a little after the dinner-hour, and just when the engineer had put on the first puffs of steam, which, disturbing the spherical state of the water, threw it against the plates of the boiler; and these being of a red heat, an explosion could not help taking place. Mr. Dunn illustrated what he meant by spherical state of the water, by a diagram representing the section of the interior of a boiler. It appears that at a certain temperature, the water of the upper stratum or surface of the boiler is driven from the sides, and held off, as it were, by wedges; gradually, and bit by bit, this separation of the water from the sides of the boiler is effected all down to the bottom of the boiler, and the water is also lifted up from that, and remains suspended, as it were, in a spherical state. The boiler plates become of a red heat; and when the water comes in contact with them, an explosion must occur—they must infallibly go.

"Mr. Dunn stated that he had received only the day before, a communication from Brussels, intimating that the opinion of the Belgian Commissioners was quite in favour of his invention. It had been tried before them, and it appeared completely to answer its exceedingly desirable end. It appears that the whole cost for twelve months will not amount to more than half-a-crown."

It is to be regretted that the Government of this country has never thought of instituting a similar commission to that of France and America. It is clear that too many experiments cannot be made on so important a subject as steam, and it has been found that each succeeding set seems in some degree to controvert what has gone before. For instance, the experiments of Southern led him to conclude, that the latent heat of steam was a constant quantity. The experiments of Watt, Clement, and Desormes seem to show that the sum of the latent and sensible heats was a constant quantity. The latest experiments which we have, were made by Regnault, at the

instance of the French Government; they seem to show that neither the law of Watt nor that of Southern is strictly correct, but that the former is considerably nearer to the truth than the latter. See *Tredgold on the Steam Engine*, for January, 1850.

It is also to be regretted, that some of the European despots have been greater patrons of science than the monarchs of this country.* Let us hope, however, that the case is altered now; the present monarch seems to strike out a different path from that of her noble ancestors. We have heard of the age of Augustus, the age of Louis XIV.; and the late illustrious Arago has stated, and justly, that we ought to have the age of Papin and Watt. Let us, then (seeing the present reign so auspiciously begun), sincerely hope that in after-times, admiring nations will celebrate with *éclat* the age of Victoria I.

Mr. Dunn, the gentleman referred to above, has invented an indicator to show the temperature of the boiler, which is described in the *Worcester Herald* in the following manner:—"The apparatus consisted of a model steam-boiler, heated by a spirit-lamp, and fitted up with the Dunn detector, or electrical indicator of temperature, which consists of a tube partially filled with mercury introduced into the interior of the boiler, so that the temperature shall unfailingly affect the mercury: a wire is let into this tube, which is connected with one pole of a galvanic battery. When the temperature rises to a height which it is prudent should be indicated, the mercury is raised in the tube, and, touching the wire, completes a galvanic circle; a stream of electricity is discharged round a piece of iron in the usual horse-shoe form, which becoming magnetic, as it is well known it will, gives motive power, and, by the same contrivance as used in the electric tele-

* It is no idle boast to say, that no other known part of this habitable globe possesses a greater degree of rational freedom than England.

graph, rings a bell. So long as the temperature remains so high as to be dangerous, the bell continues to ring, the mercury in the tube necessarily causing the completion of the galvanic circle; the bell may, in fact, be said to be the voice of the boiler plainly crying out, 'Pray look after me, for I am no longer able to take care of myself.'"

Mr. Jacob Perkins, speaking of the experiments of the Franklin Institute, says that, "from the want of some practical facts which have transpired in the course of the experiments which he has made within the last twelve years, they have, as he believed, arrived at some conclusions entirely fallacious, and which will, if not controverted, tend to injurious consequences. The most dangerous of these conclusions is, that all the destructive explosions of steam boilers have been caused by the direct pressure of the steam, and that there is no difference between the explosion and the bursting of a steam boiler. If the late Oliver Evans (who in his own department was one of the greatest men America has ever produced) had been still living, he would have given a very different opinion. Mr. Evans had a record of more than 600 burstings, before he had one explosion, although the pressure was greater when the bursting took place than when the explosion happened. I have myself witnessed enough to be perfectly satisfied that there is as much difference between a bursting and an explosion of a steam boiler, as there is between the bursting of a cannon by hydraulic pressure or by gunpowder. It is well known that a cannon bursted by hydraulic pressure is perfectly harmless, and the effect of bursting by gunpowder is also known by dreadful experience. In my endeavours to learn the nature and value of high steam, I have been so fortunate as never to have had one explosion, although I have very often had the steam up to a pressure of 100 atmospheres; but I have had more than 100 rents, or burstings. In the Adelaide Gallery, where the steam has been up daily to an average of 450 lbs. to the square inch, for more than four years, and where the exhibition of the steam gun has

been many times stopped in consequence of the bursting of the generator, yet no report has ever been heard, even by persons within a few feet of the generator. In fact, I never knew a brick to be disturbed by the bursting of the generator. About ten years since, I was exhibiting the steam gun in a large assemblage, when in the middle of a volley the steam dropped, and the balls refused to leave the barrel. I immediately ran to the furnace and found a rent about eight inches long and one inch wide in the centre of the boiler, which was three and a half feet long, seven inches diameter, and three-quarters of an inch thick, of wrought copper. I apologised to the company for stopping the experiment, as the boiler had burst; they exclaimed that it was impossible, for they had heard no explosion. My answer was, come and see, which many did, when the astonishment manifested, as well as the pleasure expressed by them, was very great. One gentleman in particular was extremely gratified at being present when the circumstance occurred, he being an owner of steam engines."

From numerous experiments, with water, alcohol, ether, and many other liquids, the following law may be deduced :—

That bodies in the spheroidal state remain constant at a temperature below that of boiling, however high the temperature of the containing vessel may be.

That water contained in boilers does pass into the spheroidal state there can be no doubt, since we know that sometimes circumstances are such that it could not possibly be otherwise, and moreover, it has actually been seen to be so. What then, are the causes which lead to this occurrence? The most obvious cause is a deficiency of water in the boiler; owing either to the negligence of the engine-man, or to some defect or derangement of the feed-pipe. When this deficiency occurs, the boiler, if the furnace underneath be in action, shortly becomes highly heated, and it is by no means an uncommon occurrence for it to reach even a red heat. If water, under these circumstances, be thrown in, the first portion becomes,

of course, spheroidal, and continues so, until, by the addition of a larger quantity, the boiler be so far cooled as to be unable to maintain the spheroidal form of the water; no sooner is this the case, than the spheroid comes into contact suddenly with the over-heated boiler, bursts into steam, and in all probability an explosion is the result.

M. Boutigny, of Evreux, who has devoted a great deal of time to the subject, has succeeded in bringing to light some most curious facts; so contrary, indeed, are some of his results to our preconceived ideas, that he confessed he should hardly have believed them possible, if he had not witnessed them. Some of these experiments he described, showing one or two of the most remarkable by way of illustration, and noticing the important consequences of this property of water, in being the frequent cause of steam-boiler explosions. It is generally stated in books, that a red or white heat is necessary in order to throw the water into this globular form. Far lower temperatures, however, are sufficient. This may be proved by throwing some water into a saucer of melted lead, a metal which melts long before it becomes luminous in the dark: the water shows no appearance of boiling, but rolls about like a little crystal ball for a considerable time.

M. Boutigny, indeed, succeeded in forming a spheroid of water in a capsule floating on oil, heated to not more than 340°, which is about 600° below what is usually called "red heat."

Water and other liquids, when in the spheroidal state, slowly and gradually disappear, though no appearance of boiling is ever observed. This is, of course, owing to slow evaporation, which goes on from every part of its surface, thus enveloping it with a film of vapour.

Of the extreme slowness of the evaporation, some opinion may be formed from the fact, which has been proved by direct experiment, that a quantity of water, which would, under ordinary circumstances, boil away at a temperature of 212° in *one minute*, will, if thrown into a vessel heated

nearly to redness, require *little less than an hour for its total dispersion.*

This was illustrated, by dropping from a glass tube three or four drops of water into a red-hot capsule of platinum, which was kept hot, and at the same time boiling about the same quantity in another capsule. The drops of water in the latter evaporated very rapidly, while those in the former became one spheroid, diminishing slowly in size, and rotating for a considerable time after the boiling water had entirely evaporated.

We have seen that when water is thrown upon a surface of red-hot platinum, it does not, as we might have expected, explode violently into steam, but, on the contrary, rolls calmly on its axis like a little world in space, and continues in the liquid state for a considerable space of time. Let us, then, endeavour to ascertain what is the temperature of the globule of water, and what relation it bears to that of the heated vessel, as well as to that of its own thin coating of vapour.

Having again formed a spheroid in the same manner as before mentioned, the bulb of a delicate thermometer was plunged into it, taking care that it did not come in contact with the heated metal. The temperature thus indicated was invariably 205°.

Perhaps one of the most curious facts which have been established in connection with the subject, is, that any variation in the temperature of a vessel containing a spheroid, does not affect the temperature of the spheroid itself. Thus, it is found that a spheroid of water, when contained in a crucible heated considerably below redness, is just *as hot* as one contained in a crucible intensely heated to whiteness in the most powerful blast furnace*.

* From Papers read by Mr. Bowman, Mr. Fairbairn in the Chair, at the Manchester Institute, and published in the "Engineer and Mechanic's Journal."

Specific Heat.

The specific heat, or the comparative capacity of bodies of equal weight to receive heat, varies widely. Thus, if 1 lb. of mercury at 32° be mixed with water at 62°, the temperature will become 61°; or, if the mercury had been 62° and the water 32°, the common temperature would have been 33°, showing that the capacity of mercury for heat is about $\frac{1}{30}$ of that of water. It may, therefore, be considered as the ratio of the heat in a given weight or volume to those of the standard body. Iron shows a specific heat of ·113 or $\frac{1}{9}$ that of water, and steam ·847. Water is usually made the standard of comparison for ponderous bodies, and air for gaseous bodies. The capacity of bodies for heat is also tested by the quantity of ice they will melt; thus, equal weights of iron and lead, heated to 100°, would melt 11 grains by the iron and only 3 grains by the lead, each falling to 95°. The same test applied to fuel has given the following results:—

1 lb. of	good coal	melts	90 lbs.	of ice.
,,	coke	,,	84 lbs.	,,
,,	wood	,,	32 lbs.	,,
,,	wood charcoal	,,	95 lbs.	,,
,,	peat	,,	18 lbs.	,,

The following Table contains the specific heat of different bodies:—

Specific Heat. Specific Heat of Water = 1.		Relative Weight of Atoms. Oxygen = 1.	Products of the Atomic Weight by the Specific Heat.
Bismuth	0·0288	13·30	0·3830
Lead	0·0293	12·95	0·3794
Gold	0·0298	12·43	0·3704
Platinum	0·0314	11·16	0·3740
Tin	0·0514	7·35	0·3779
Silver	0·0557	6·75	0·3759
Zinc	0·0927	4·03	0·3736
Tellurium	0·0912	4·03	0·3675
Copper	0·0949	3·957	0·3755
Nickel	0·1035	3·69	0·3819
Iron	0·1100	3·392	0·3731
Cobalt	0·1498	2·46	0·3685
Sulphur	0·1880	2·011	0·3780

APPENDIX.

On Gudgeons.

In gudgeons, one-fifth of the diameter is usually allowed for wear, and on this principle Mr. Tredgold gives the following

Rule.—Multiply the stress in pounds by the length in inches, and the cube root of the product, divided by 9, is the diameter of the gudgeon in inches.

Example.—If the stress of a gudgeon be 10 tons, and its length 7 inches, what is the diameter?

$$10 \text{ tons} = 22400 \text{ lbs.},$$

$$7 \times 22400 = 156800,$$

$$\sqrt{156800} = 54 \text{ nearly},$$

and, $\frac{54}{9} = 6$ inches, the diameter required.

On the Forms of Beams.

In the construction of beams it is necessary that their form should be such that they will be equally strong throughout; or, in other words, that they will offer an equal resistance to fracture in all their parts, and will, therefore, be equally liable to break at one part of their length as at another.

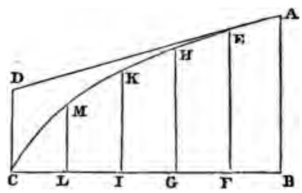
If a beam be fixed at one end and loaded at the other, and the breadth uniform throughout its length, then, that the beam may be equally strong throughout, its form must be that of a parabola.

This form is generally used in the beams of steam

engines; and, in double-acting steam engines the beam is strained sometimes from one side, and sometimes from the other, therefore both the sides should be of the same form.

The crank as used in the steam engine should be of the same form.

Dr. Young and Mr. Tredgold have considered that it will answer better in practice, to have some straight-lined figure to include the parabolic form, and the form which they propose is, to draw a tangent to the point A of the parabola ACB. But, as few practical men understand how to draw a tangent to a parabola, or even a parabola itself, we will here show how they may do both.



We will, in the first place, show how to draw a parabola.

Let CB represent the length of the beam, and AB the semi-ordinate, or half the base; then, by the property of the parabola, the squares of all ordinates to the same diameter are to one another as their respective abscissas.

Now, if we take $CB = 4$ feet, and $AB = 1$ foot, we may proceed to apply this property to determine the length of the semi-ordinates corresponding to every foot in the length of the beam, as follows:—

$$CB : CF :: AB^2 : EF^2,$$

that is, $48 : 36 :: (12)^2 : 108 = EF^2$,

the square root of which is 10·4 nearly $= EF$.

And, $$CB : CG :: AB^2 : GH^2,$$

$$48 : 24 :: (12)^2 : 72 = GH^2,$$

the square root of which is 8·5 nearly $= GH$.

$$CB : CI :: AB^2 : IK^2,$$

$$48 : 12 :: (12)^2 : 36 = IK^2,$$

the square root of which is 6 inches $= IK$.

Now, if we take $CL = 6$ inches,

$$\text{then, } CB : CL :: AB^2 : LM^2,$$

$$48 : 6 :: (12)^2 : 18 = LM^2,$$

the square root of which is 4·24, which is very near $4\frac{1}{4}$ inches $= LM$.

Now, if any flexible rod be bent so as just to touch the tops A, E, H, K, M of the ordinates, and the vertex C, then the form of this rod is a parabola.

To draw a tangent to any point A of a parabola:—

From the vertex C of the parabola draw CD perpendicular to CB, and make it equal to $\frac{1}{2}$ AB; then join A and D, and the right line AD will be a tangent to the parabola at the point A; that is, it touches the parabola at that point.

In the same manner we may draw a tangent to the parabola at any other point, by raising a perpendicular at the vertex equal to half the semi-ordinate at that point.

Beams of Pumping Engines.

If a beam be fixed at one end and the load applied at the other, or, which is the same thing, if a beam be supported on a centre of motion, then, as we have said before, the figure of equal strength is a parabola, the breadth of the beam being the same throughout.

If the beam be fixed in the middle, the following rule may be used:—

Rule.—Multiply the cube of the length of the beam in feet from the centre of motion, by the weight in pounds; and twice this product, divided by 2662 times the deflection, will give the product of the breadth and cube of the depth in inches.

This rule is for an uniform beam, of which the section is a rectangle.

But if the beam be of a parabolic kind, divide by 1628 instead of 2662.

Example.—Let it be required to determine the breadth and depth of a beam in the form of a parabola, for a pumping engine, its whole length being 24 feet, the parts on each side of the centre of motion equal, and the straining force 28000 lbs.; the deflection not to exceed $\frac{1}{4}$ of an inch.

Then, by the rule,

$\frac{28000 \times 12^3 \times 2}{1628 \times \frac{1}{4}} = 237759 =$ the product of the breadth and cube of the depth.

If we take the breadth $= 3$ inches, then

$$\frac{237759}{3} = 79253,$$

$$\sqrt[3]{79253} = 43 \text{ inches nearly} = \text{depth}.$$

If the beam had been uniform and its section a rectangle,

$$\frac{28000 \times 12^3 \times 2}{2662 \times \frac{1}{4}} = 145406.$$

If the breadth be made 3 inches, then,

$$\frac{145406}{3} = 48468 \text{ nearly},$$

$$\sqrt[3]{48468} = 36{\cdot}5 \text{ nearly} = \text{depth required}.$$

CRANKS.

Rule.—Multiply the weight or power, in pounds, acting at the end of the crank, by the cube of its length in feet; and this product, divided by 2662 times the deflection, will give the product of the breadth and cube of the depth in inches.

Example.—If the force acting upon a crank be 6000 lbs., and its length 3 feet, what will be its breadth and depth, so that the deflection may not exceed $\frac{1}{10}$th of an inch?

By the rule, $\frac{6000 \times 3^3}{2662 \times \cdot 1} = 609$ nearly $=$ breadth multiplied by the cube of the depth.

If the breadth be made $= 3$ inches, then

$$\frac{609}{3} = 203,$$

$$\sqrt[3]{203} = 6 \text{ inches nearly} = \text{depth.}$$

If the depth at the end where the force acts be half the depth at the axis, divide by 1628 instead of 2662:

$\frac{6000 \times 3^3}{1628 \times \cdot 1} = 995 =$ breadth multiplied by the cube of the depth.

If the breadth be 3 inches, then

$$\frac{995}{3} = 331 \cdot 6,$$

$$\sqrt[3]{331 \cdot 6} = 6 \cdot 92 \text{ inches, the depth required.}$$

Wheels.

Rule.—Multiply the weight or power, in pounds, acting at the end of the arm, by the cube of its length in feet; and this product divided by 2662 times the number of arms multiplied by the deflection, will give the product of the breadth and cube of the depth.

When the depth at the rim is half that at the axis, divide by 1628 instead of 2662.

Example.—If the force which acts at the circumference of a spur wheel be 1600 lbs., the radius of the wheel 6 feet, and the number of arms 8; the deflection not to exceed $\frac{1}{10}$ of an inch; required the breadth and depth.

Then, $1600 \times 6^3 = 345600$

$$\frac{345600}{2662 \times 8 \times \cdot 1} = 162 \cdot 28 = \text{breadth and cube of the depth.}$$

If the breadth be made 2·5 inches, then

$$\frac{162{\cdot}28}{2{\cdot}5} = 64{\cdot}9 \text{ nearly,}$$

$$\text{and, } \sqrt[3]{64{\cdot}9} = 4{\cdot}019, \text{ the depth.}$$

Friction of an Axle.

Let P be the pressure on the bearings.

Rule.—Multiply this pressure by f, the ratio of the friction to the pressure corresponding to the bodies in contact; this product will give the friction $P.f$. Multiply this friction by the space passed over in one revolution, or by the circumference $2\pi r = 6{\cdot}28 . r$ of the axis; the product, $6{\cdot}28 . P . f . r$, is the work expended in each revolution.

This product multiplied by the number of revolutions per second, will give the work on friction per second; *i. e.*

$$6{\cdot}28 . P . f . r . n = \text{work of friction per second.}$$

Example.—Given the radius of the axis $= 6$ inches, the weight of the shaft and other parts pressing on it $= 1000$ lbs.; the shaft making 10 revolutions per minute. What is the work expended on friction?

f, the coefficient of friction in the tables, is ·07;

$$Pf = {\cdot}07 \times 1000 = 70 \text{ lbs.};$$

the space passed over by the circumference of the axis is

$$\frac{6{\cdot}28 \times {\cdot}5 \times 10}{60} = {\cdot}523 \text{ feet per second};$$

hence the quantity of work expended on friction is

$$70 \times {\cdot}523 = 36{\cdot}61 \text{ lbs.}$$

Friction of a Pivot.

Rule.—Multiply the pressure P by the ratio f of the friction to the pressure; this product will give the friction.

Multiply this product by two-thirds of the exterior circumference of the base of the pivot, or by $4\cdot19r$.

The product, $4\cdot19f.P.r$, will be the work expended in each revolution by the friction of the pivot.

To have the work expended per second, multiply the above product by the number of revolutions per second.

Hence $4\cdot19.n.f.P.r$, is the work expended on friction, n being the number of revolutions per second.

Example.—Given the radius of the base of the pivot = 6 inches, the weight pressing upon it = 1000 lbs.; the pivot making 10 revolutions per minute. What is the work expended on friction?

$$f, \text{ the coefficient in the tables, is } \cdot07;$$

$$P.f = \cdot07 \times 1000 = 70 \text{ lbs.};$$

the space passed over by the circumference of the base is

$$\frac{4\cdot19 \times \cdot5 \times 10}{60} = \cdot349 \text{ feet per second};$$

hence the quantity of work expended on friction is

$$70 \times \cdot349 = 24\cdot43 \text{ lbs.}$$

Friction of Cylinders.

The proportion which the friction of a large cylinder bears to the friction of any number, the sum of the areas of which is equal to the area of the large cylinder, may be shown as follows:—

Let d = the diameter of one of the small cylinders,
n = the number of them.

Then, $$d^2.n\frac{\pi^*}{4} = \text{area of large cylinder};$$

$$\therefore\ d\sqrt{n} = \text{diameter of large cylinder.}$$

But the friction is proportional to the circumference of

* π stands for the circular measure 3·14159.

the cylinder; therefore, the friction of the small cylinders may be represented by

$$\pi d n,$$

and the friction of the large cylinder by

$$\pi d \sqrt{n}.$$

Hence,

the friction of the large cylinder : the friction of all the small cylinders $:: \pi d \sqrt{n} : \pi d n;$

$$:: \sqrt{n} : n,$$

which, in the case of 4 small cylinders, becomes 1 : 2; that is, the friction of 4 cylinders is double the friction of 1 cylinder, the area of which is equal to the sum of the areas of all the 4.

On Boilers.

Mr. Millington says, that a boiler for 20 horse-power is usually 15 feet long and 6 wide, therefore 90 feet of surface, or 4½ feet to 1 horse-power; a boiler for a 14 horse-power, 60 feet of surface, or 4·3 feet to 1 horse-power. But engineers generally allow 5 feet of surface to 1 horse-power; and Mr. Hicks, of Bolton, proportions his boilers at the rate of 5½ square feet of horizontal surface of water to each horse-power.

Mr. Watt allows 25 cubic feet of space to each horse-power.

Length of Boilers for Locomotive Engines.

In the Northern and Eastern Counties Railway, the length of the boiler is 8 feet; while in the North Midland Counties Railway, in the Great Western Railway, and in the Hartlepool Railway, the length of the boiler is 8½ feet. In Stephenson's locomotive engines the length of the boiler is between 11 and 12 feet.

In the Belgium railways the length of the boiler is 8 feet 2 inches.

In the Bordeaux and La Teste Railway the length of the boiler is 8 feet 9 inches.

And in America the length varies from 10 to 14 feet.

To find the inside Diameter of a Locomotive Boiler.

Rule.—Multiply the diameter of the cylinder in inches by 3·11. The product is the inside diameter of the boiler in inches.

Example.—Required the inside diameter of the boiler for a locomotive engine, the diameter of the cylinder being 15 inches.

Then according to the rule,

$$15 \times 3{\cdot}11 = 46{\cdot}65 \text{ inches}$$

which is the inside diameter of the boiler required.

Diameter of the Steam Dome inside.

It is obvious that the diameter of the steam dome may be varied considerably, according to circumstances; but the first indication is to make it large enough. It is usual, however, in practice, to proportion the diameter of the steam dome to the diameter of the cylinder, and there appears to be no great objection to this. The following rule will be found to give the diameter of the dome, usually adopted in practice.

Rule.—Multiply the diameter of the cylinder in inches by 1·43. The product is the diameter of the dome in inches.

Example.—Required the diameter of the steam dome for a locomotive engine, the diameter of the cylinders being 15 inches.

Then by the rule,

$$15 \times 1{\cdot}43 = 21{\cdot}45 \text{ inches} = \text{diameter of dome required.}$$

Height of Steam Dome.

The height of the steam dome may vary. Judging from practice, it appears that a uniform height of 2½ feet would answer very well.

Area of Fire-grate.

The following rule determines the area of the fire-grate, usually given in practice. We may remark, that the area of the fire-grate in practice follows a more certain rule than any other part of the engine appears to do; but it is in all cases much too small, and occasions a great loss of power by the urging of the blast it renders necessary, and a rapid deterioration of the furnace plates from excessive heat. There is no good reason why the furnace should not be nearly as long as the boiler; it would then resemble the furnace of a marine boiler, and be as manageable.

Rule.—Multiply the diameter of the cylinder in inches by ·77. The product is the area of the fire-grate in superficial feet.

Example.—Required the area of the fire-grate of a locomotive engine, the diameter of the cylinders being 15 inches.

Then according to the rule,

$$\cdot 77 \times 15 = 11 \cdot 55 \text{ square feet, area of fire-grate.}$$

Though this rule represents the usual practice, the area of the fire-grate should not depend upon the size of the cylinder, but upon the quantity of the steam to be generated.

Steam Pipes.

The internal diameter of the steam pipe is usually rather more than one-fifth of the diameter of the steam cylinder. The area of the passages through valves, in some of Watt's engines, are nearly 1 square inch to each horse-power. This is, in some cases, too large for steam passages, but rather too small for the exhausting-valve passages. Indeed, the larger the exhausting-valve passages are, the better*.

* The only proper way of proportioning the steam passages is, by taking into consideration the velocity of the steam through them.

Air-Pump.

The proportion of the air-pump, as given by Mr. Watt, is usually about two-thirds of the diameter of the cylinder, when the length of the stroke of the air-bucket is half the length of the stroke of the steam-piston.

The area of the passages between the condenser and the air-pump should never be less than one-fourth of the area of the air-pump. The apertures through the air-bucket should have the same proportion; and if it be convenient, the discharging flap or valve should be made larger.

The capacity of the condenser should at least be equal to that of the air-pump; but, where convenience will admit of it, the larger it is, the better.

On the Lead of the Slide.

The theories of notes on the philosophy of engineering, which I propose to continue from time to time in this Journal *, will be commenced with a somewhat bold design, which is no less than to call into question certain of the views of the Comte de Pambour, on the Theory of the Steam Engine. The extraordinary perspicuity of his investigations, the admirable arrangement of his experiments,

Mr. Tredgold says, the force of steam in the boiler, multiplied by the velocity and the area of the passage, must be equal to the elastic force on the piston multiplied by its area and velocity. That is,

$$f \cdot a \cdot v = p A V.$$

Where f is the force of the steam in the boiler in inches of mercury, a the area of the steam passages, v the velocity, p the force on the piston in inches of mercury, A its area, and V its velocity;

$$\therefore\ v = \frac{p \cdot A \cdot V}{f \cdot a},$$

$$\text{or,} \qquad a = \frac{p \cdot A \cdot V}{f \cdot v}.$$

* From the "Civil Engineer and Architect's Journal," by Mr. Homersham Cox.

and the beautiful simplicity of his physical conceptions, have procured for him so just a reputation amongst those whose applause is really worth having, that I almost despair of being able to overcome the prejudice which will exist against any attack upon his doctrines. Nevertheless, the following tenets respecting the lead of the slide appear to me, after careful consideration, so essentially erroneous, that I think that he himself, on reconsideration, would hardly be prepared to defend them.

"We have already mentioned the advantages arising from the lead of the slide, with regard to the play and conservation of the engine; but there is another advantage no less important, resulting from this disposition, namely, that of obtaining a greater velocity; and, consequently, a greater useful effect of the engine with a given load.

"This effect is easy to comprehend, for if the suppression of the steam from the boiler, instead of being made precisely at the end of the stroke of the piston, takes place, for instance, at the moment when the piston is yet an inch from the bottom of the cylinder, from that moment steam ceases to flow into the cylinder. Thus, with regard to the quantity of steam admitted into the cylinder or expended at each stroke of the piston, the length of the stroke is really diminished by an inch. Now, it is the quantity of steam produced by the boiler which regulates and limits the velocity of the engine. Suppose that such production furnished m cylinders-full of steam per minute, when the total length l of the stroke was filled with steam; now, no more than the length $l - a$ is filled with steam; the same production, then, will fill per minute a number of cylinders expressed by $m \times \frac{l}{l - a}$. Hence, in fine, the velocity of the engine will be increased in the inverse ratio of the lengths of the cylinders which are filled with steam." See *Treatise on Locomotive Engines,* Chap. 16, Sec. 2.

The error in this passage appears to me to be this, that M. de Pambour, in comparing the two cases, supposes that

the density of the steam in the cylinder is the same in either instance. Now, were this so, we should arrive at this strange conclusion—that if the same motive force, which in the first instance acted through a distance l, be made to act through a shorter distance $l - a$, it will move its load with *greater* velocity. This appears to me perfectly inadmissible, and I shall show that the motive force is in the second case so altered that, though it act through a shorter distance, it does the same work that the original motive force does in acting through the whole length of the cylinder.

It need scarcely be said, that I here assume the truth of Boyle or Mariotte's law; it is ascertained from M. de Pambour's experiments, that this law is really true for the steam in high-pressure engines, when the densities are not greatly varied.

It is also presupposed that the reader is aware, that when an engine is in motion, the density of steam in the cylinder may vary considerably from that in the boiler. For if the boiler generate such and such a number of cubic feet of steam per minute, of a given pressure, and if that steam have, owing to the rapidity of the stroke, to occupy twice as many cubic feet in the cylinder, by Mariotte's law the cylinder-pressure would be only half the boiler-pressure.

Before, however, examining *what* the cylinder-pressure must be, that as much work may be done where there is as where there is not a lead of the slide, I must draw attention to another passage from M. de Pambour's treatise.

"At the moment when the piston reaches the point which corresponds to the *lead of the slide for the suppression of the steam, the motive force is suppressed;* and *when the piston, continuing its stroke in virtue of its acquired velocity,* arrives at the point which corresponds to the lead for the admission of the steam, it not only receives no further impulse in the direction of the motion, but suffers an opposition from the motive force itself, then let in against it."

The passages which I have marked by italics, I may unhesitatingly affirm to be erroneous. The motive force is *not* suppressed when the steam is cut off, for it continues to act expansively; the piston does *not* continue its motion in virtue of its required velocity merely, but does receive "further impulse in the direction of the motion."

The last clause of the quotation refers to the *lead of admission*, but I shall in the following investigation consider the effects of the lead of suppression alone, as being the most important, and because I do not wish to complicate the question with effects which are of such a nature that they may be considered separately.

In estimating the work done by steam of a given pressure π, in a cylinder of given length α, for a given lead of the slide, I will take the usual measure of the "travail," or "work done," namely, the pressure multiplied by the space through which it is exerted.

Suppose the piston has risen to a height h in the cylinder at the moment when the steam is cut off, then up to that moment the full pressure π has been exerted through a distance h, or the work done $= \pi h$.

Let x be a point to which the piston has risen after the suppression of the steam, the pressure is now decreased in the inverse proportion of the spaces occupied, or of $x : h$, and if we suppose this pressure constant for a distance dx, the work done at this point is

$$\frac{h}{x} \pi \, dx.$$

And the whole work done after the steam is cut off is determined by integrating this expression between the limits h, the point of suppression, and a, the height of the cylinder. Effecting this integration and adding the work done before suppression, we have for the whole amount of work done,

$$\pi h + \pi h \log a - \pi h \log h \qquad \text{(A.)}$$

I will just pause here for a moment to observe, that the

expression which I have marked (A) vanishes (as it ought) when h is put $= 0$, or when the steam is wholly suppressed. The first two members of (A) obviously vanish when h is zero; the last one, however, becomes $\pi \, 0 \log 0$, which is an illusive expression; but if for $\log h$ we use its expansion

$$(h-1) - \tfrac{1}{2}(h-1)^2 + \tfrac{1}{3}(h-1)^3 - \ldots\ldots$$

and multiply this by h, it is seen that the whole vanishes when $h = 0$.

Suppose that when the engine is working at uniform velocity, that is, is at its normal state of motion, the resistance offered to the piston by the work to be done is p lbs. to the square inch, then of course the work to be done in each stroke $= pa$, if a be the height of the cylinder; and if steam be admitted during the whole stroke, it is clear that the pressure of steam in the cylinder also must be p lbs. to the square inch. If, however, steam be admitted during part only of the stroke, the pressure must be so much increased that the work done by the steam may still be the same. We have then to equate our expression, obtained above, with pa. Hence

$$pa = \pi h\,(1 + \log a - \log h).$$

Here, then, nothing is more easy than to compare the pressures in the two cases where there is, and where there is not, a lead of the slide; for the equation gives at once

$$\pi = p \cdot \frac{a}{h\,(1 + \log a - \log h)}. \qquad \text{(B.)}$$

This, then, must be the pressure of the steam supplied to the cylinder, in order that when cut off at height h, it may do the same work that steam of pressure p would if acting through the whole length of the cylinder. By examining equation B, it will be seen that when $h = 0$, or the steam is totally suppressed, the expression for the corresponding pressure becomes $= \infty$, as it ought. But it is carefully to be noted, that the real limit to the value

of this pressure arises from the consideration that it can never be greater than in the boiler. We must, therefore, never give a value which would make the value of π greater than the boiler-pressure. If we did, the engine would no longer be able to do the work we have assigned to it.

I will now resume the expression (A) and employ it for the purpose of comparing a steam engine which has, with a steam engine which has not, a lead of the slide. I think the following will be a very convenient way of instituting this comparison. There are four principal conditions under which the problem of the steam engine is varied.

I. The degree of pressure in the boiler.

II. The rapidity of evaporation.

III. The load moved.

IV. The velocity with which the load is moved.

The reader will see, that of these conditions the first two are of the nature of causes, the last two are effects. Now, the way in which I propose to examine the result of introducing a lead of the slide into an engine which was before worked at full pressure, is this; to see what change it would produce in *each* of the above two effects, supposing the other three conditions to remain unaltered. That is, to see

1st. How the velocity must be changed if I., II., and III. remain unchanged.

2nd. How the load must be changed if I., II., and IV. remain unchanged.

I do not mean to say that the lead of the slide might not be made to produce changes in both effects at once, but the subject will be exhibited most clearly by the plan suggested of considering each change separately.

On the alteration of the velocity produced by a lead of the slide.

1. To examine the alteration in the velocity produced by a lead in the slide, the rapidity of evaporation being the

same, and the load of resistance remaining unchanged. Suppose the evaporation such that it would produce per minute m cylinders-full of steam of the pressure p; then it would produce $2m$ cylinders-full of the pressure $\frac{1}{2}p$, $3m$ cylinders-full of the density $\frac{1}{3}p$, and, for the same reason, $\frac{h}{a}(1 + \log a - \log h)\,m$ cylinders-full of pressure $\frac{a}{h(1 + \log a - \log h)\,p}$. If, however, the steam be cut off when it has filled the cylinder to a height h, instead of a, the number of times the cylinder is filled per minute will be increased in the proportion $a : h$. Hence, finally, the number of cylinders-full of steam, and, therefore, the number of strokes per minute, are defined in reference to (B) by the expression

$$\frac{a\,h}{h\,a}(1 + \log a - \log h)\,m,$$

or the velocity will be increased in the ratio

$$1 : 1 + \log a - \log h. \qquad \text{(C.)}$$

Hence, the smaller the value of h, the greater the velocity, subject only to this limit, that the value of h must never be such as would suppose a higher pressure in the cylinder than that in the boiler. We obtain, therefore, the following practical rules.

The highest velocity for a given load and given evaporation is obtained by cutting off the steam at such a point that the steam in the cylinder shall, during admission, have the same pressure as the steam in the boiler.

The velocity will be increased by increasing the boiler-pressure and the lead of the slide conjointly.

The rules which are here demonstrated are of the highest importance, and I confess that it is with no little satisfaction that I arrive at results which I did not foresee till the very moment of interpreting the analytical formula which I have here exhibited. These rules, even when viewed apart from the analysis which has led to them,

bear the highest marks of probability; and they have this advantage, that they are not merely theoretically true, but correspond to the actual working condition of steam engines, and require no practical modifications arising from friction of machinery and other unknown resistances.

Before quitting the subject, it will be well to see what is the value of the lead of the slide which gives the cylinder-pressure equal to the boiler-pressure, and, therefore, as we have shown, produces the greatest velocity.

Let the effective pressure of the boiler be P lbs. to the square inch, and, as before, let the load offer a resistance of p lbs. on each square inch of the piston. Then, we have from equation (B) for the maximum value of h,

$$\frac{p}{P} = \frac{h}{a}\left(1 + \log \frac{a}{h}\right).$$

From this equation we can easily find what relation to the load and the boiler-pressure the lead of the slide should have for a maximum velocity. For instance, let us suppose the resistance to the piston 48 lbs. per inch, and the boiler-pressure 50 lbs. per inch. In this case, it will be found on trial that the equation is nearly satisfied by putting $h = \frac{3}{4} a$, for we have

$$\frac{p}{P} = \frac{3}{4}\left(1 + \log \frac{4}{3}\right)$$
$$= \frac{3}{4}\left(1 + \cdot 2876\right) = \cdot 96 \text{ nearly.}$$

This would give the relation of p to P equal to 48 : 50, or, conversely, if the effective boiler-pressure were 50 lbs. per inch and the resistance 48 lbs., the maximum velocity for a given evaporation (and therefore for a given amount of fuel) would be obtained by cutting off the steam at $\frac{3}{4}$ths the stroke.

" Suppose, in effect, that a load of 50 tons gross, tender included, be drawn up a plane inclined $\frac{1}{500}$, by an engine with two cylinders 11 inches in diameter, stroke of the piston 16 inches, wheels 6 feet, friction 103 lbs., total pressure of the steam in the boiler 65 lbs., or effective pressure 50 lbs. per square inch.

"We have already found above, that the total resistance opposed by that load to the motion of the piston, in the case of this engine, is 48 lbs. per square inch."

I have taken this extract from M. de Pambour's account of his admirably-conducted experiments, to show that the case I have supposed accords with practice. It appears that in this case, with an evaporation of one cubic foot of water per minute, the engine would move the train at the rate of 20·7 miles per hour, there being no lead of the slide. The lead being $\frac{3}{4}$ths, we find from (C) that the velocity (for the same evaporation) would be increased in the ratio

$$1 : 1 + \log \tfrac{3}{4}, \text{ or, } 1 : 1{\cdot}2876,$$

or the velocity would be increased from 20·7 to 26·64 miles an hour—no trifling advantage certainly. It is necessary, however, to refer the reader to another extract from M. de Pambour's work, because the considerations which it offers apply to the investigation here made, and are essential to its accuracy.

"It is necessary here to remark, that as this lead offers a resistance precisely equal to the pressure of the steam in the boiler, and as we have seen that at the moment of starting of every engine, the power must necessarily exert an effort greater than the resistance, it would be impossible for the engine to set itself in motion with the load. If, then, we would make the engine work with this load, it is understood that the aid of another engine would be requisite to start it; or else the engine-man must for a few minutes close the safety-valve, to create in the boiler a sufficient excess of pressure, till the uniform motion be attained. Then the momentary excess of pressure may be withdrawn, and the engine will continue its motion without any external aid.

"However, as on railways there continually occur little inequalities or accidental imperfections in the road, and as the engine ought to be capable of overcoming them, it is not to be expected that it can be made to perform an entire

trip, working precisely at its maximum of useful effect, or with its maximum load. The preceding determination, therefore, is to be considered only as showing what the engine may perform on arriving with a velocity already acquired, at an inclined plane situated at a certain point of the line, or as indicating the point towards which our aim should tend as much as possible, in order to accomplish producing the maximum of useful effect, but without reckoning on obtaining it completely in practice.

"We here neglect the little necessary difference between the pressures in the cylinder and in the boiler, from the flowing of the steam from the one vessel to the other. It plainly tends somewhat to reduce the load of the engine, increasing in a corresponding manner the velocity of maximum useful effect."

I ought perhaps, to add, that by improvements which have been (I believe) invented since M. de Pambour wrote, the expansion gearing is placed under the control of the engine-driver, so that he can regulate the lead of the slide while the engine is in motion. The slide need not, therefore, have any lead till the engine has attained its full velocity.

On the alteration in the load produced by a lead of the slide.

2. I come now to consider what alteration must be made in the load or resistance for a lead of the slide, supposing the evaporation and the velocity to remain unchanged. It will be clear, that as far as concerns locomotive engines, this second inquiry principally affects luggage trains, while the former inquiry as to the means of increasing velocity most affects passenger trains.

Suppose, as before, that when there is no lead of slide the boiler supplies m cylinders-full of steam of the pressure p. If, now, we suppose the velocity unchanged, there will still be the same number of strokes per minute, and the quantity of steam supplied for each stroke will also be the same as before, only it will fill the cylinder to a height

h instead of a, its pressure will therefore be increased in the proportion $a : h$. Now, by the principles already laid down, the work done by steam of this pressure $\frac{a}{h}p$ used expansively is

$$\frac{a}{h}p \cdot h\,(1 + \log a - \log h), \text{ or } ap\,(1 + \log a - \log h).$$

When, however, the steam was used without expansion, the total work done in each stroke was ap. We have, therefore, to multiply by the quantity within the bracket to get the increased effect for expansion. It is clear also, that the increased load or resistance, since it acts through a distance a, is found by dividing by a, that is, the increased load, which we call p', is expressed by the equation

$$p' = p\,(1 + \log a - \log h),$$

or the load for a given amount of fuel and a given velocity is increased by the lead of the slide, in the proportion

$$1 : 1 + \log a - \log h. \qquad \text{(D.)}$$

The greater then will be the load which can be moved, as the value of h becomes smaller; subject only to a limit similar to that in the first question. Since the pressure in the cylinder can never be greater than that in the boiler, the maximum effect will be produced by giving h such a value that

$$\frac{a}{h}p = P.$$

Hence we have for the greatest useful value of h,

$$h = \frac{p}{P}a,$$

where the evaporation is equivalent to m cylinders-full of steam of the pressure p per minute. Here p is an unknown quantity, but m is known, because the velocity of the engine is supposed to be known. By ascertaining, therefore, the cubical content of the cylinder, and referring to

M. de Pambour's tables, for the relation of the volume of steam of any pressure to the water from which it is generated, we shall be able to ascertain p.

To take an instance, let us suppose the evaporation of an engine such as that which M. de Pambour takes as the average, namely, that $\frac{3}{4}$ cubic foot of water is transferred to the cylinder in steam each minute. Also, let the number of revolutions of the wheel per minute be 116. (This corresponds nearly to a speed of 20 miles an hour, the driving wheel being 6 feet over.) The engine fills each of its cylinders twice in each revolution: the number of cylinders-full of steam per minute will therefore be 464. Suppose the capacity of each cylinder $\frac{9}{10}$ cubic foot. Then "m cylinders-full of steam of pressure p" become equivalent to 417·6 cubic feet of steam of pressure p. But this steam is supplied by $\frac{3}{4}$ cubic foot of water. The steam would have, therefore, 556 times the volume of the water which produced it; and by the tables it appears that this relative volume corresponds to a pressure of 50 lbs. per inch, or 35 lbs. effective pressure, which is therefore the value of p. Hence, if the effective boiler pressure be 50 lbs. per inch, the formula

$$h = \frac{p}{P} a \text{ becomes } h = \frac{35}{50} a,$$

or the greatest useful lead of the slide is in this case given by cutting off the steam at $\frac{7}{10}$ths of the stroke.

It will be seen from (D) that the load would in this case be increased in the proportion

$$1 : 1 + \log \tfrac{7}{10}, \text{ or } 1 : 1·356,$$

or for the same amount of fuel and the same velocity the load may be increased about $\frac{7}{20}$ths by the lead of the slide.

On the Screw.

The surface of the screw is supposed to be generated by a line, normal to the axis, moving along the axis and revolving round the axis with uniform velocities, to the extent of one revolution.

Let v denote the velocity of the vessel in feet per second;

ω, the angular velocity of the screw;

h, the length, or pitch of the screw;

r, the radius of any point P;

α, the angle of inclination of the surface of the screw at that point, with a plane section perpendicular to the axis; and let

$$k = \frac{h}{2\pi} \dots\dots (1).$$

$$u = k\omega = \frac{h}{2\pi}\omega \dots\dots (2).$$

Then, $$\tan\alpha = \frac{h}{2r\pi} = \frac{k}{r} \dots\dots (3).$$

Effective angular velocity

$$= \frac{r\omega\tan\alpha - v}{\tan\alpha} = \frac{r}{k}(k\omega - v) = \frac{r}{k}(u - v).$$

Velocity perpendicular to the screw

$$= \left(r\omega - v\frac{\cos\alpha}{\sin\alpha}\right)\sin\alpha$$

$$= (r\omega\tan\alpha - v)\cos\alpha = (u - v)\cos\alpha.$$

Elementary surface at $P = \dfrac{2\pi r}{\cos\alpha}dr$;

$\therefore$ the elementary pressure

$$dp = \tfrac{1}{2}\cdot\frac{W}{g}\cdot\frac{2\pi r}{\cos\alpha}dr(u - v)^2\cos^2\alpha$$

$$= \tfrac{1}{2}\cdot\frac{W}{g}\cdot 2\pi r dr\cos\alpha(u - v)^2.$$

Let $C = \frac{1}{2} . \frac{W}{g} . 2\pi (u - v)^2$ . . . .(4),

and $dp = Crdr \cos\alpha = \frac{C r^2 dr}{\sqrt{k^2 + r^2}}$ (by 3);

$\therefore$ d (effective power) $= v\,(dp \cos\alpha) = Cv \frac{r^3 dr}{k^2 + r^2}$

d (full power) $= \omega r\,(dp \sin\alpha) = Cu \frac{r^3 dr}{k^2 + r^2}$.

$$\text{But} \int \frac{r^3 dr}{k^2 + r^2} = \int \left(r dr - \frac{k^2 r dr}{k^2 + r^2} \right)$$

$$= \frac{1}{2} \left(r^2 - k^2 \log. \frac{k^2 + r^2}{k^2} \right).$$

Let, therefore, $A = r^2 - k^2 \log. \frac{k^2 + r^2}{k^2}$ . . . . (5),

$$\left.\begin{array}{ll} \text{the effective power} & = \frac{1}{2} CA \times v \\ \text{the full power} & = \frac{1}{2} CA \times u \end{array}\right\} \ldots\ldots\ldots (6).$$

Multiply by $\frac{60}{33000} = \frac{1}{550}$ and

the effective horse power $= \frac{1}{1100} CA \times v$.

the full horse power $= \frac{1}{1100} CA \times u$.

Make $c = \frac{1}{2} . \frac{W}{g} \times \frac{\pi}{550} = \cdot 0056894$

$\log c = 7 \cdot 75507$ . . . . (7),

$$\left.\begin{array}{ll} \text{the effective horse power} & = cA\,(u - v)^2 \times v \\ \text{the full horse power} & = cA\,(u - v)^2 \times u \end{array}\right\} \ldots\ldots (8).$$

Again, by (6) the effective propelling pressure

$$= \frac{1}{2} CA = \frac{1}{2} . \frac{W}{g} \pi A\,(u - v)^2.$$

But if $S =$ the effective surface of resistance of the vessel, we shall have the effective resistance $= \frac{1}{2} \cdot \frac{W}{g} \cdot S v^2$;

$$\therefore\ S v^2 = \pi A (u - v)^2$$

$$\left(\frac{u}{v} - 1\right)^2 = \frac{S}{\pi A},$$

$$\left.\begin{aligned} u &= v\left(1 + \sqrt{\frac{S}{\pi A}}\right) \\ \text{Revolutions per second} &= \frac{u}{h} \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (9).$$

Should the propelling surface of the screw be generated between two limiting values r, r_1, it will be requisite to estimate the function A, equation (5), between those limits, viz.:—

$$A = \left(r^2 - k^2 \log. \frac{k^2 + r^2}{k^2}\right)$$
$$\sim \left(r_1^2 - k^2 \log. \frac{k^2 + r_1^2}{k^2}\right) \ldots\ldots (10).$$

The proportion of the power of the engine usefully effective in the propulsion of the vessel

$$= \frac{\text{effective power}}{\text{full power}} = \frac{v}{u} = \frac{1}{1 + \sqrt{\frac{S}{\pi A}}}.$$

Note. $$S = \frac{2{\cdot}5\ (\text{tonnage})^{\frac{2}{3}}}{15} = \frac{1}{6}\ (\text{tonnage})^{\frac{2}{3}}$$

$$\text{or,}\ S = \frac{3\ (\text{tonnage})^{\frac{2}{3}}}{15} = \frac{1}{5}\ (\text{tonnage})^{\frac{2}{3}}$$

according to the form of the vessel.

That is, "in absence of better information," says Mr.

Woolhouse, "the effective resisting surface of the vessel may be roughly estimated at about

$$\frac{1}{5} \text{ or } \frac{1}{6} \, (\textit{tonnage})^{\frac{2}{3}},$$

according to the construction of the vessel."

"The larger the diameter and the less the pitch of the screw, the greater will be the proportion of the effective power on the vessel."

On Chimneys.

From numerous experiments by Péclet, Combes, Girard, Daubuisson, and others, the following laws are deduced in reference to the production of the motion of air, or gases in pipes, or close galleries:—

1st. All other things being the same, the pressure required to generate velocity only, is proportional to the square of such velocity.

2nd. All other things being the same, the pressure required to overcome the resistance offered by the walls, or sides of the gallery, is also proportional to the square of the velocity with which the air moves.

Whence it follows, that all other things being equal, whatever may be the pressure and velocity causing ventilation, a certain fixed proportion, or percentage, is expended in generating the velocity, and the remaining constant proportion is expended in overcoming the frictional resistance by the walls of the chimney.

From the experiments of Magnus, it would appear, that air expands 0·36651 of its volume at 32° Fahrenheit, by being heated from that temperature to 212° on the same scale, whilst those of Regnault give 0·36650 as the increase, agreeing so nearly that the fraction

$$\frac{0{\cdot}3665}{212^\circ - 32^\circ} = \frac{1}{491},$$

is adopted as more correct than that of $\frac{1}{480}$, which has been so frequently used to express the fractional part of the volume of air at 32°, by which its volume is increased by each degree of Fahrenheit.

Now, if by v we represent the volume of any given weight of air, at the temperature 32°; by v', the volume it assumes on being heated to any higher temperature t; and by v'', the volume it assumes on being heated to any third temperature t'; then we have

$$v' : v'' :: v\left(1 + \frac{t - 32}{491}\right) : v . \left(1 + \frac{t' - 32}{491}\right)$$

and hence the equation,

$$v''\left(1 + \frac{t - 32}{491}\right) = v'\left(1 + \frac{t' - 32}{491}\right)$$

from which

$$v'' = v'\left(\frac{t' + 491 - 32}{t + 491 - 32}\right) = v'\left(\frac{t' + 459}{t + 459}\right) \text{ .... (1)},$$

an equation from which the volume of air at any given temperature can readily be determined from its volume at any other temperature.

It is known, that if by h we represent the height of a chimney, in which air is allowed to expand freely by heat under a constant pressure (barometrical); by heating the air in such a chimney from a temperature t to a higher temperature t', its contents would be expanded in the ratio of

$$h \text{ to } h\left(\frac{459 + t'}{459 + t}\right);$$

and hence the difference,

$$h\left(\frac{459 + t'}{459 + t}\right) - h, \text{ or, } h\left\{\frac{459 + t'}{459 + t} - 1\right\} \text{ .... (2)}$$

would represent the length of a column of air at t', having the same area as the chimney, which would be expelled from it by such an increase of temperature. The height of such expelled column reduced to air of the temperature t would be

$$\frac{h\left\{\frac{459+t'}{459+t}-1\right\}}{\frac{459+t'}{459+t}}=\frac{h\,(t'-t)}{459+t'} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3).$$

This may be called the *motive height*,

$$v=8\sqrt{\textit{Motive height}}=8\sqrt{h\left(\frac{t'-t}{459+t'}\right)}\quad\ldots\ldots\ldots (4),$$

which is the same theorem as that given by Tredgold, in his work on *Warming and Ventilating Buildings*, excepting the constant, as before observed.

On the Steam Jet.

It is by means of a jet of high pressure steam introduced into the chimney of locomotives, that the draught necessary for the enormous consumption of fuel in them has been attained. This has led to its application to mines and to manufactories.

As may be gathered from Mr. Mather's recent treatise on "*The Coal Mines, their Dangers, and Means of Safety*," the application of the steam jet has been successful; a better, cheaper, and more equable ventilation than by the old system, having been obtained*. As to

* Peclet, in his most valuable *Traité de la Chaleur, considérée dans ses Applications*, Art. 577, and following, says:—

"This mode of draught, which is at the same time economical and of extreme simplicity, since only one boiler is required, is often preferred to all others, inasmuch, as by using it, the draught may be

the results obtained by its use in manufactories, we are indebted to the same gentleman for the following exposition of them.

At Messrs. Swinbourne and Co.'s Alkali Works, in South Shields, a boiler of 30-horse power, pressed to about 30 lbs. on the inch, projects the steam through six jets, each ¼ of an inch in diameter, ventilating six furnaces in which sea salt for making carbonate of soda is decomposed by sulphuric acid. A great heat being required, the force of these jets readily excite it by freely supplying

varied more easily than by making use of machines, the effect of which cannot increase but by an augmentation of the velocity, which is sometimes inconvenient and very limited. But in order to produce a powerful momentary draught, both the boiler in action and the spare one may be made to work together; and besides, the means may be procured for both, of producing more steam by burning more fuel, although less usefully.

"It would be important to bring the steam to the exhausting-ports by a pipe of a large diameter, in order that in its travel the steam should experience but little friction; and, also, that the exhausting ports should have the proper dimensions, so that the steam at its issue should nearly acquire the velocity which corresponds to the pressure in the boiler.

"It would also be very important to introduce the steam into the chimney by several jets, so placed as to fill the whole chimney at a small distance from the orifices with the dilated steam. It is easily conceived, that it is indispensable for the steam to act on the whole section, otherwise the air would come back from top to bottom in that part of the chimney on which the steam was not acting directly.

"It has been pretended, that in the steam jet, all the effect produced by the *vis viva* of the steam was not obtained, unless the air was at a temperature sufficiently high for no condensation of the steam to take place; but I do not think that this circumstance bears any very sensible influence; however, it would be useful to try some experiments on that subject.

"It has also been pretended, from experiments made on locomotives, that by the use of an intermittent jet, a greater useful effect was produced than by a continuous one; supposing this to be true, it should be seen if, in the experiments that have been made, the tension of the steam was not greater with the intermittent jet than by the continuous one."

atmospheric air, and the muriatic acid comes off in large quantities, which, if allowed to escape into the atmosphere, would be very deleterious both to life and vegetation.

By the power of the jets, this gas is drawn into an horizontal chimney, which runs along from the furnace till it comes to a square perpendicular tower, 25 feet high, and which is divided down the middle by a partition, and filled with broken coke. This tower has water discharged into its top, at the rate of about 20 gallons per minute. The jets not only draw the gas and smoke along the horizontal chimney, but up to the top of the tower through the stream of falling water, then down the other side of the partition, and thence through as many cones as jets (each cone being 1 foot long, 9 inches diameter at the base and 4 inches at the apex), into a passage in which they are inserted, and which runs along like a drain under the surface of the ground for about 100 yards.

The object that the Messrs. Swinbourne had in view when they introduced the steam jet into their works, was to condense the noxious muriatic acid gas, and at the same time to ventilate the furnaces in the most perfect manner. So much is this done, and so perfectly, that a chimney about 200 feet high is entirely superseded for these purposes. When this chimney was used, the imperfect ventilation of the furnaces and condensation of the gases were such, that it frequently cost the works £300 a year, for damages to vegetation, caused by these gases.

At the Don Alkali Works, on the Tyne, a boiler, 20 feet by 5, with a pressure of 30 lbs. to the inch, its steam driven through jets $\frac{3}{16}$ths of an inch, or thereabouts, directed through cones whose apexes are only 4 inches in diameter, produces a white heat in the furnaces, and draws along through the horizontal passages and then up a tower 40 feet high, filled with coke and falling water, the whole smoke and muriatic acid gas. The distance is not less than 150 yards before they reach the place where the jets are acting. The gas is thus condensed to the amount of nearly 50 tons a week, and,

instead of going to destroy vegetation, as it formerly did to the cost of the manufacturers, when chimneys for ventilation were used, it now goes to form bleaching powder, and is one of the most valuable, instead of being one of the most injurious products of the manufactory.

The deleterious *hydrochloric acid*, driven off in the above chemical process in such abundance, requires to be disposed of without touching animal or vegetable life. If it be thrown into the atmosphere at certain elevations, its specific gravity being upwards of one-fourth more than that of the air, it descends upon the earth in large volumes, affecting life and vegetation. As water absorbs nearly 500 times its own bulk of this gas, the steam jets, in the above processes, force the gas, by a powerful ventilation, up through the large descending column of water in the condensing towers. The absorption in this operation is nearly complete, and what may pass free is absorbed by the steam, after it passes the jets in the horizontal chimneys. The exit in the yard of the manufactory scarcely shows, in the escapage, the slightest vestige of the acid.

The chimneys, which the manufacturers of alkali have had erected sometimes 400 feet high, at a cost of from £2000 to £3000, as in the Tyne works, may for the future be saved by the adoption of the steam jet, which does its work not only more economically, but more efficiently than those costly fabrics.

This process, effected by the steam jet, is suggestive of horizontal chimneys for manufactories, for the purpose of preventing smoke or other obnoxious effluvia from vitiating the atmosphere of towns; and, also, for the ventilation of town sewers and the destruction of their gases.

One of the northern mines, possessing nearly as many miles of passages, and filled with as deadly gases as the sewers of London, is kept perfectly free from these gases, and in a state fitted for the daily operations of the miners, by the steam jet.

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
1·01	·0099503	1·41	·3435897	1·81	·5933268
1·02	·0198026	1·42	·3506568	1·82	·5988365
1·03	·0295588	1·43	·3576744	1·83	·6043159
1·04	·0392207	1·44	·3646431	1·84	·6097655
1·05	·0487902	1·45	·3715635	1·85	·6151856
1·06	·0582689	1·46	·3784364	1·86	·6205764
1·07	·0676586	1·47	·3852624	1·87	·6259384
1·08	·0769610	1·48	·3920420	1·88	·6312717
1·09	·0861777	1·49	·3987761	1·89	·6365768
1·10	·0953102	1·50	·4054651	1·90	·6418538
1·11	·1043600	1·51	·4121096	1·91	·6471032
1·12	·1133287	1·52	·4187103	1·92	·6523251
1·13	·1222176	1·53	·4252677	1·93	·6575200
1·14	·1310283	1·54	·4317824	1·94	·6626879
1·15	·1397619	1·55	·4382549	1·95	·6678293
1·16	·1484200	1·56	·4446858	1·96	·6729444
1·17	·1570037	1·57	·4510756	1·97	·6780335
1·18	·1655144	1·58	·4574248	1·98	·6830968
1·19	·1739533	1·59	·4637340	1·99	·6881346
1·20	·1823215	1·60	·4700036	2·00	·6931472
1·21	·1906203	1·61	·4762341	2·01	·6981347
1·22	·1988508	1·62	·4824261	2·02	·7030974
1·23	·2070141	1·63	·4885800	2·03	·7080357
1·24	·2151113	1·64	·4946962	2·04	·7129497
1·25	·2231435	1·65	·5007752	2·05	·7178397
1·26	·2311117	1·66	·5068175	2·06	·7227059
1·27	·2390169	1·67	·5128236	2·07	·7275485
1·28	·2468600	1·68	·5187937	2·08	·7323678
1·29	·2546422	1·69	·5247285	2·09	·7371640
1·30	·2623642	1·70	·5306282	2·10	·7419373
1·31	·2700271	1·71	·5364933	2·11	·7466879
1·32	·2776317	1·72	·5423242	2·12	·7514160
1·33	·2851789	1·73	·5481214	2·13	·7561219
1·34	·2926696	1·74	·5538851	2·14	·7608058
1·35	·3001045	1·75	·5596157	2·15	·7654678
1·36	·3074846	1·76	·5653138	2·16	·7701082
1·37	·3148107	1·77	·5709795	2·17	·7747271
1·38	·3220834	1·78	·5766133	2·18	·7793248
1·39	·3293037	1·79	·5822156	2·19	·7839015
1·40	·3364722	1·80	·5877866	2·20	·7884573

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
2·21	·7929925	2·61	·9593502	3·01	1·1019400
2·22	·7975071	2·62	·9631743	3·02	1·1052568
2·23	·8020015	2·63	·9669838	3·03	1·1085626
2·24	·8064758	2·64	·9707789	3·04	1·1118575
2·25	·8109302	2·65	·9745596	3·05	1·1151415
2·26	·8153648	2·66	·9783261	3·06	1·1184149
2·27	·8197798	2·67	·9820784	3·07	1·1216775
2·28	·8241754	2·68	·9858167	3·08	1·1249295
2·29	·8285518	2·69	·9895411	3·09	1·1281710
2·30	·8329091	2·70	·9932517	3·10	1·1314021
2·31	·8372475	2·71	·9969486	3·11	1·1346227
2·32	·8415671	2·72	1·0006318	3·12	1·1378330
2·33	·8458682	2·73	1·0043015	3·13	1·1410330
2·34	·8501509	2·74	1·0079579	3·14	1·1442227
2·35	·8544153	2·75	1·0116008	3·15	1·1474024
2·36	·8586616	2·76	1·0152306	3·16	1·1505720
2·37	·8628899	2·77	1·0188473	3·17	1·1537315
2·38	·8671004	2·78	1·0224509	3·18	1·1568811
2·39	·8712933	2·79	1·0260415	3·19	1·1600209
2·40	·8754687	2·80	1·0296194	3·20	1·1631508
2·41	·8796267	2·81	1·0331844	3·21	1·1662709
2·42	·8837675	2·82	1·0367368	3·22	1·1693813
2·43	·8878912	2·83	1·0402766	3·23	1·1724821
2·44	·8919980	2·84	1·0438040	3·24	1·1755733
2·45	·8960880	2·85	1·0473189	3·25	1·1786549
2·46	·9001613	2·86	1·0508216	3·26	1·1817271
2·47	·9042181	2·87	1·0543120	3·27	1·1847899
2·48	·9082585	2·88	1·0577902	3·28	1·1878434
2·49	·9122826	2·89	1·0612564	3·29	1·1908875
2·50	·9162907	2·90	1·0647107	3·30	1·1939224
2·51	·9202827	2·91	1·0681530	3·31	1·1969481
2·52	·9242589	2·92	1·0715836	3·32	1·1999647
2·53	·9282193	2·93	1·0750024	3·33	1·2029722
2·54	·9321640	2·94	1·0784095	3·34	1·2059707
2·55	·9360933	2·95	1·0818051	3·35	1·2089603
2·56	·9400072	2·96	1·0851892	3·36	1·2119409
2·57	·9439058	2·97	1·0885619	3·37	1·2149127
2·58	·9477893	2·98	1·0919233	3·38	1·2178757
2·59	·9516578	2·99	1·0952733	3·39	1·2208299
2·60	9555114	3·00	1·0986123	3·40	1·2237754

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
3·41	1·2267122	3·81	1·3376291	4·21	1·4374626
3·42	1·2296405	3·82	1·3402504	4·22	1·4398351
3·43	1·2325605	3·83	1·3428648	4·23	1·4422020
3·44	1·2354714	3·84	1·3454723	4·24	1·4445632
3·45	1·2383742	3·85	1·3480731	4·25	1·4469189
3·46	1·2412685	3·86	1·3506671	4·26	1·4492691
3·47	1·2441545	3·87	1·3532544	4·27	1·4516138
3·48	1·2470322	3·88	1·3558351	4·28	1·4539530
3·49	1·2499017	3·89	1·3584091	4·29	1·4562867
3·50	1·2527629	3·90	1·3609765	4·30	1·4586149
3·51	1·2556160	3·91	1·3635373	4·31	1·4609379
3·52	1·2584609	3·92	1·3660916	4·32	1·4632553
3·53	1·2612978	3·93	1·3686394	4·33	1·4655675
3·54	1·2641266	3·94	1·3711807	4·34	1·4678743
3·55	1·2669475	3·95	1·3737156	4·35	1·4701758
3·56	1·2697605	3·96	1·3762440	4·36	1·4724720
3·57	1·2725655	3·97	1·3787661	4·37	1·4747630
3·58	1·2753627	3·98	1·3812818	4·38	1·4770487
3·59	1·2781521	3·99	1·3837912	4·39	1·4793292
3·60	1·2809338	4·00	1·3862943	4·40	1·4816045
3·61	1·2837077	4·01	1·3887912	4·41	1·4838746
3·62	1·2864740	4·02	1·3912818	4·42	1·4861396
3·63	1·2892326	4·03	1·3937663	4·43	1·4883995
3·64	1·2919836	4·04	1·3962446	4·44	1·4906543
3·65	1·2947271	4·05	1·3987168	4·45	1·4929040
3·66	1·2974631	4·06	1·4011829	4·46	1·4951487
3·67	1·3001916	4·07	1·4036429	4·47	1·4973883
3·68	1·3029127	4·08	1·4060969	4·48	1·4996230
3·69	1·3056264	4·09	1·4085449	4·49	1·5018527
3·70	1·3083328	4·10	1·4109869	4·50	1·5040774
3·71	1·3110318	4·11	1·4134230	4·51	1·5062971
3·72	1·3137236	4·12	1·4158531	4·52	1·5085119
3·73	1·3164082	4·13	1·4182774	4·53	1·5107219
3·74	1·3190856	4·14	1·4206957	4·54	1·5129269
3·75	1·3217558	4·15	1·4231083	4·55	1·5151272
3·76	1·3244189	4·16	1·4255150	4·56	1·5173226
3·77	1·3270749	4·17	1·4279160	4·57	1·5195132
3·78	1·3297240	4·18	1·4303112	4·58	1·5216990
3·79	1·3323660	4·19	1·4327007	4·59	1·5238800
3·80	1·3350010	4·20	1·4350845	4·60	1·5260563

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
4·61	1·5282278	5·01	1·6114359	5·41	1·6882491
4·62	1·5303947	5·02	1·6134300	5·42	1·6900958
4·63	1·5325568	5·03	1·6154200	5·43	1·6919391
4·64	1·5347143	5·04	1·6174060	5·44	1·6937790
4·65	1·5368672	5·05	1·6193882	5·45	1·6956155
4·66	1·5390154	5·06	1·6213664	5·46	1·6974487
4·67	1·5411590	5·07	1·6233408	5·47	1·6992786
4·68	1·5432981	5·08	1·6253112	5·48	1·7011051
4·69	1·5454325	5·09	1·6272778	5·49	1·7029282
4·70	1·5475625	5·10	1·6292405	5·50	1·7047481
4·71	1·5496879	5·11	1·6311994	5·51	1·7065646
4·72	1·5518087	5·12	1·6331544	5·52	1·7083778
4·73	1·5539252	5·13	1·6351056	5·53	1·7101878
4·74	1·5560371	5·14	1·6370530	5·54	1·7119944
4·75	1·5581446	5·15	1·6389967	5·55	1·7137979
4·76	1·5602476	5·16	1·6409365	5·56	1·7155981
4·77	1·5623462	5·17	1·6428726	5·57	1·7173950
4·78	1·5644405	5·18	1·6448050	5·58	1·7191887
4·79	1·5665304	5·19	1·6467336	5·59	1·7209792
4·80	1·5686159	5·20	1·6486586	5·60	1·7227666
4·81	1·5706971	5·21	1·6505798	5·61	1·7245507
4·82	1·5727739	5·22	1·6524974	5·62	1·7263316
4·83	1·5748464	5·23	1·6544112	5·63	1·7281094
4·84	1·5769147	5·24	1·6563214	5·64	1·7298840
4·85	1·5789787	5·25	1·6582280	5·65	1·7316555
4·86	1·5810384	5·26	1·6601310	5·66	1·7334238
4·87	1·5830939	5·27	1·6620303	5·67	1·7351891
4·88	1·5851452	5·28	1·6639260	5·68	1·7369512
4·89	1·5871923	5·29	1·6658182	5·69	1·7387102
4·90	1·5892352	5·30	1·6677068	5·70	1·7404661
4·91	1·5912739	5·31	1·6695918	5·71	1·7422189
4·92	1·5933085	5·32	1·6714733	5·72	1·7439687
4·93	1·5953389	5·33	1·6733512	5·73	1·7457155
4·94	1·5973653	5·34	1·6752256	5·74	1·7474591
4·95	1·5993875	5·35	1·6770965	5·75	1·7491998
4·96	1·6014057	5·36	1·6789639	5·76	1·7509374
4·97	1·6034198	5·37	1·6808278	5·77	1·7526720
4·98	1·6054298	5·38	1·6826882	5·78	1·7544036
4·99	1·6074358	5·39	1·6845453	5·79	1·7561323
5·00	1·6094379	5·40	1·6863989	5·80	1·7578579

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
5·81	1·7595805	6·21	1·8261608	6·61	1·8885837
5·82	1·7613002	6·22	1·8277699	6·62	1·8900954
5·83	1·7630170	6·23	1·8293763	6·63	1·8916048
5·84	1·7647308	6·24	1·8309801	6·64	1·8931119
5·85	1·7664416	6·25	1·8325814	6·65	1·8946168
5·86	1·7681496	6·26	1·8341801	6·66	1·8961194
5·87	1·7698546	6·27	1·8357763	6·67	1·8976198
5·88	1·7715567	6·28	1·8373699	6·68	1·8991179
5·89	1·7732559	6·29	1·8389610	6·69	1·9006138
5·90	1·7749523	6·30	1·8405496	6·70	1·9021075
5·91	1·7766458	6·31	1·8421356	6·71	1·9035989
5·92	1·7783364	6·32	1·8437191	6·72	1·9050881
5·93	1·7800242	6·33	1·8453002	6·73	1·9065751
5·94	1·7817091	6·34	1·8468787	6·74	1·9080600
5·95	1·7833912	6·35	1·8484547	6·75	1·9095425
5·96	1·7850704	6·36	1·8500283	6·76	1·9110228
5·97	1·7867469	6·37	1·8515994	6·77	1·9125011
5·98	1·7884205	6·38	1·8531680	6·78	1·9139771
5·99	1·7900914	6·39	1·8547342	6·79	1·9154509
6·00	1·7917594	6·40	1·8562979	6·80	1·9169226
6·01	1·7934247	6·41	1·8578592	6·81	1·9183921
6·02	1·7950872	6·42	1·8594181	6·82	1·9198594
6·03	1·7967470	6·43	1·8609745	6·83	1·9213247
6·04	1·7984040	6·44	1·8625285	6·84	1·9227877
6·05	1·8000582	6·45	1·8640801	6·85	1·9242486
6·06	1·8017098	6·46	1·8656293	6·86	1·9257074
6·07	1·8033586	6·47	1·8671761	6·87	1·9271641
6·08	1·8050047	6·48	1·8687205	6·88	1·9286186
6·09	1·8066481	6·49	1·8702625	6·89	1·9300710
6·10	1·8082887	6·50	1·8718021	6·90	1·9315214
6·11	1·8099267	6·51	1·8733394	6·91	1·9329696
6·12	1·8115621	6·52	1·8748743	6·92	1·9344157
6·13	1·8131947	6·53	1·8764069	6·93	1·9358598
6·14	1·8148247	6·54	1·8779371	6·94	1·9373017
6·15	1·8164520	6·55	1·8794650	6·95	1·9387416
6·16	1·8180767	6·56	1·8809906	6·96	1·9401794
6·17	1·8196988	6·57	1·8825138	6·97	1·9416152
6·18	1·8213182	6·58	1·8840347	6·98	1·9430489
6·19	1·8229351	6·59	1·8855533	6·99	1·9444805
6·20	1·8245493	6·60	1·8870696	7·00	1·9459101

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
7·01	1·9473376	7·41	2·0028305	7·81	2·0554049
7·02	1·9487632	7·42	2·0041790	7·82	2·0566845
7·03	1·9501866	7·43	2·0055258	7·83	2·0579624
7·04	1·9516080	7·44	2·0068708	7·84	2·0592388
7·05	1·9530275	7·45	2·0082140	7·85	2·0605135
7·06	1·9544449	7·46	2·0095553	7·86	2·0617866
7·07	1·9558604	7·47	2·0108949	7·87	2·0630580
7·08	1·9572739	7·48	2·0122327	7·88	2·0643278
7·09	1·9586853	7·49	2·0135687	7·89	2·0655961
7·10	1·9600947	7·50	2·0149030	7·90	2·0668627
7·11	1·9615022	7·51	2·0162354	7·91	2·0681277
7·12	1·9629077	7·52	2·0175661	7·92	2·0693911
7·13	1·9643112	7·53	2·0188950	7·93	2·0706530
7·14	1·9657127	7·54	2·0202221	7·94	2·0719132
7·15	1·9671123	7·55	2·0215475	7·95	2·0731719
7·16	1·9685099	7·56	2·0228711	7·96	2·0744290
7·17	1·9699056	7·57	2·0241929	7·97	2·0756845
7·18	1·9712993	7·58	2·0255131	7·98	2·0769384
7·19	1·9726911	7·59	2·0268315	7·99	2·0781907
7·20	1·9740810	7·60	2·0281482	8·00	2·0794415
7·21	1·9754689	7·61	2·0294631	8·01	2·0806907
7·22	1·9768549	7·62	2·0307763	8·02	2·0819384
7·23	1·9782390	7·63	2·0320878	8·03	2·0831845
7·24	1·9796212	7·64	2·0333976	8·04	2·0844290
7·25	1·9810014	7·65	2·0347056	8·05	2·0856720
7·26	1·9823798	7·66	2·0360119	8·06	2·0869135
7·27	1·9837562	7·67	2·0373166	8·07	2·0881534
7·28	1·9851308	7·68	2·0386195	8·08	2·0893918
7·29	1·9865035	7·69	2·0399207	8·09	2·0906287
7·30	1·9878743	7·70	2·0412203	8·10	2·0918640
7·31	1·9892432	7·71	2·0425181	8·11	2·0930984
7·32	1·9906103	7·72	2·0438143	8·12	2·0943306
7·33	1·9919754	7·73	2·0451088	8·13	2·0955613
7·34	1·9933387	7·74	2·0464016	8·14	2·0967905
7·35	1·9947002	7·75	2·0476928	8·15	2·0980182
7·36	1·9960599	7·76	2·0489823	8·16	2·0992444
7·37	1·9974177	7·77	2·0502701	8·17	2·1004691
7·38	1·9987736	7·78	2·0515563	8·18	2·1016923
7·39	2·0001278	7·79	2·0528408	8·19	2·1029140
7·40	2·0014800	7·80	2·0541237	8·20	2·1041341

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
8·21	2·1053529	8·61	2·1529243	9·01	2·1983350
8·22	2·1065702	8·62	2·1540851	9·02	2·1994443
8·23	2·1077861	8·63	2·1552445	9·03	2·2005523
8·24	2·1089998	8·64	2·1564026	9·04	2·2016591
8·25	2·1102128	8·65	2·1575593	9·05	2·2027647
8·26	2·1114243	8·66	2·1587147	9·06	2·2038691
8·27	2·1126343	8·67	2·1598687	9·07	2·2049722
8·28	2·1138428	8·68	2·1610215	9·08	2·2060741
8·29	2·1150499	8·69	2·1621729	9·09	2·2071748
8·30	2·1162555	8·70	2·1633230	9·10	2·2082744
8·31	2·1174596	8·71	2·1644718	9·11	2·2093727
8·32	2·1186622	8·72	2·1656192	9·12	2·2104697
8·33	2·1198634	8·73	2·1667653	9·13	2·2115656
8·34	2·1210632	8·74	2·1679101	9·14	2·2126603
8·35	2·1222615	8·75	2·1690536	9·15	2·2137538
8·36	2·1234584	8·76	2·1701959	9·16	2·2148461
8·37	2·1246539	8·77	2·1713367	9·17	2·2159372
8·38	2·1258479	8·78	2·1724763	9·18	2·2170272
8·39	2·1270405	8·79	2·1736146	9·19	2·2181160
8·40	2·1282317	8·80	2·1747517	9·20	2·2192034
8·41	2·1294214	8·81	2·1758874	9·21	2·2202898
8·42	2·1306098	8·82	2·1770218	9·22	2·2213750
8·43	2·1317967	8·83	2·1781550	9·23	2·2224590
8·44	2·1329822	8·84	2·1792868	9·24	2·2235418
8·45	2·1341664	8·85	2·1804174	9·25	2·2246235
8·46	2·1353491	8·86	2·1815467	9·26	2·2257040
8·47	2·1365304	8·87	2·1826747	9·27	2·2267833
8·48	2·1377104	8·88	2·1838015	9·28	2·2278615
8·49	2·1388889	8·89	2·1849270	9·29	2·2289385
8·50	2·1400661	8·90	2·1860512	9·30	2·2300144
8·51	2·1412419	8·91	2·1871742	9·31	2·2310890
8·52	2·1424163	8·92	2·1882959	9·32	2·2321626
8·53	2·1435893	8·93	2·1894163	9·33	2·2332350
8·54	2·1447609	8·94	2·1905355	9·34	2·2343062
8·55	2·1459312	8·95	2·1916535	9·35	2·2353763
8·56	2·1471001	8·96	2·1927702	9·36	2·2364452
8·57	2·1482676	8·97	2·1938856	9·37	2·2375130
8·58	2·1494339	8·98	2·1949998	9·38	2·2385797
8·59	2·1505987	8·99	2·1961128	9·39	2·2396452
8·60	2·1517622	9·00	2·1972245	9·40	2·2407096

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
9·41	2·2417729	9·61	2·2628042	9·81	2·2834022
9·42	2·2428350	9·62	2·2638442	9·82	2·2844211
9·43	2·2438960	9·63	2·2648832	9·83	2·2854389
9·44	2·2449559	9·64	2·2659211	9·84	2·2864556
9·45	2·2460147	9·65	2·2669579	9·85	2·2874714
9·46	2·2470723	9·66	2·2679936	9·86	2·2884861
9·47	2·2481288	9·67	2·2690282	9·87	2·2894998
9·48	2·2491843	9·68	2·2700618	9·88	2·2905124
9·49	2·2502386	9·69	2·2710944	9·89	2·2915241
9·50	2·2512917	9·70	2·2721258	9·90	2·2925347
9·51	2·2523438	9·71	2·2731562	9·91	2·2635443
9·52	2·2533948	9·72	2·2741856	9·92	2·2945529
9·53	2·2544446	9·73	2·2752138	9·93	2·2955604
9·54	2·2554934	9·74	2·2762411	9·94	2·2965670
9·55	2·2565411	9·75	2·2772673	9·95	2·2975725
9·56	2·2575877	9·76	2·2782924	9·96	2·2985770
9·57	2·2586332	9·77	2·2793165	9·97	2·2995806
9·58	2·2596776	9·78	2·2803395	9·98	2·3005831
9·59	2·2607209	9·79	2·2813614	9·99	2·3015846
9·60	2·2617631	9·80	2·2823823	10·00	2·3025851

PARALLEL MOTION TABLES,

When the Length of the Stroke is not taken into consideration.

Radius of Beam in Inches.	Length of Parallel Bar in Inches.	Length of Radius Rod in Inches.	Radius of Beam in Inches.	Length of Parallel Bar in Inches.	Length of Radius Rod in Inches.
72	18	162	78	27	96·5
—	21	123·9	—	30	76·8
—	24	96	—	33	61·36
—	27	75	—	36	49
—	30	58·8	—	39	39
—	33	46	—	42	30·857
—	36	36	—	45	24·2
—	39	27·9	—	48	18·75
—	42	21·428	—	51	14·3
—	45	16·2	—	54	10·666
—	48	12	—	57	7·7
—	51	8·6	—	60	5·4
—	54	6	84	18	242
78	18	200	—	21	189
—	21	154·7	—	24	150
—	24	121·5	—	27	120·3

Radius of Beam in Inches.	Length of Parallel Bar in Inches.	Length of Radius Rod in Inches.	Radius of Beam in Inches.	Length of Parallel Bar in Inches.	Length of Radius Rod in Inches.
114	63	41·3	132	72	50
—	66	34·9	—	75	43·32
—	69	29·3	—	78	37·384
—	72	24·5	—	81	32·111
—	75	20·3	—	84	27·428
120	42	144·8	—	87	23·27
—	45	125	—	90	19·6
—	48	108	—	93	16·35
—	51	93·35	138	48	168·75
—	54	80·666	—	51	148·4
—	57	69·6	—	54	130·666
—	60	60	—	57	115
—	63	51·5	—	60	101·4
—	66	44·181	—	63	89·3
—	69	37·7	—	66	78·545
—	72	32	—	69	69
—	75	27	—	72	60·5
—	78	22·6	—	75	52·9
—	81	18·77	—	78	46·153
126	42	168	—	81	40
—	45	145·8	—	84	34·643
—	48	126·75	—	87	30
—	51	110·3	—	90	25·6
—	54	96	—	93	21·7
—	57	83·5	144	48	192
—	60	72·6	—	51	169·6
—	63	63	—	54	150
—	66	54·545	—	57	132·7
—	69	47	—	60	117·6
—	72	40·5	—	63	104·14
—	75	34·7	—	66	92·181
—	78	29·33	—	69	81·5
—	81	25	—	72	72
—	84	21	—	75	63·5
—	87	17·48	—	78	55·846
132	48	147	—	81	49
—	51	128·6	—	84	42·857
—	54	112·666	—	87	37·35
—	57	98·7	—	90	32·4
—	60	86·4	—	93	28
—	63	75·5	—	96	24
—	66	66	—	99	20·4
—	69	57·5			

Radius of Beam in Inches.	Length of Parallel Bar in Inches.	Length of Radius Rod in Inches.	Radius of Beam in Inches.	Length of Parallel Bar in Inches.	Length of Radius Rod in Inches.
84	30	97·2	96	57	26·7
—	33	78·8	—	60	21·6
—	36	64	—	63	17·3
—	39	52	—	66	13·636
—	42	42	—	69	10·5
—	45	33·3	—	72	8
—	48	27	—	75	5·9
—	51	21·35	102	30	172·8
—	54	16·666	—	33	144·3
—	57	12·8	—	36	121
—	60	9·6	—	39	101·7
—	63	7	—	42	85·738
—	66	4·9	—	45	72·2
90	18	288	—	48	60·75
—	21	226·7	—	51	51
—	24	181·5	—	54	42·666
—	27	147	—	57	35·5
—	30	120	—	60	29·4
—	33	98·4	—	63	24·14
—	36	81	—	66	19·636
—	39	66·7	—	69	15·8
—	42	54·857	—	72	12·5
—	45	45	108	36	144
—	48	36·75	—	39	122
—	51	29·9	—	42	103·716
—	54	24	—	45	88·2
—	57	19	—	48	75
—	60	15	—	51	63·7
—	63	11·5	—	54	54
—	66	8·727	—	57	45·6
—	69	6·4	—	60	38·4
—	72	4·5	—	63	32·14
96	24	216	—	66	26·727
—	27	176·3	—	69	22
—	30	145·2	—	72	18
—	33	120·3	—	75	14·52
—	36	100	114	42	123·428
—	39	83·3	—	45	105·8
—	42	69·4	—	48	90·75
—	45	57·8	—	51	77·8
—	48	48	—	54	66·666
—	51	39·7	—	57	57
—	54	32·666	—	60	48·6

Table *of* **Plane Surfaces,** *when they have been some time in contact.*

Surfaces in contact.	Disposition of the Fibres.	State of the Surfaces.	Co-efficient of Friction.
Experiments of M. Morin.			
Oak upon oak	parallel	without unguent	0·62
	ditto	rubbed with dry soap	0·44
	perpendicular	without ungent	0·54
	ditto	with water	0·71
	endways of one upon the flat surface of the other	without unguent	0·43
Oak upon elm	parallel	ditto	0·38
Elm upon oak	ditto	ditto	0·69
	ditto	rubbed with dry soap	0·41
	perpendicular	without unguent	0·57
Ash, fir, beech, service-tree, upon oak . . .	parallel	without unguent	0·53
Tanned leather upon oak	the leather flat	ditto	0·61
	the leather length-ways, but	ditto	0·43
	sideways	steeped in water	0·79
Black dressed leather, or strap leather. upon a plane surface of oak . . .	parallel	without unguent	0·74
Black dressed leather, or strap leather. upon a rounded surface of oak . .	perpendicular	ditto	0·47
Hemp matting upon oak	parallel	ditto	0·50
	ditto	steeped in water	0·87
Hemp cords upon oak .	ditto	without unguent	0·80
Iron upon oak	ditto	ditto	0·62
		steeped in water	0·65
Cast-iron upon oak . .	ditto	ditto	0·65
Copper upon oak . . .	ditto	without unguent	0·62

TABLE *of* PLANE SURFACES—(*continued.*)

Surfaces in contact.	Disposition of the Fibres.	State of the Surfaces.	Co-efficient of Friction.
EXPERIMENTS OF M. MORIN —*continued.*			
Ox-hide as a piston-sheath upon cast-iron	flat or side-ways	steeped in water	0·62
		with oil, tallow, or hog's lard	0·12
Black dressed leather, or strap leather, upon a cast-iron pulley . .	flat	without unguent	0·28
		steeped	0·38
Cast-iron upon cast-iron	ditto	without unguent	0·16
Iron upon cast-iron . .	ditto	ditto	0·19
Oak, elm, yoke elm, iron, cast-iron and brass, sliding two and two one upon another .	ditto	with tallow	0·10
		with oil, or hog's lard	0·15
Calcareous oolite stone upon calcareous oolite	ditto	without unguent	0·74
Hard calcareous stone, called muschelkalk, upon calcareous oolite	ditto	ditto	0·75
Brick upon calcareous oolite	ditto	ditto	0·67
Oak upon calcareous oolite	wood end-ways	ditto	0·63
Iron upon calcareous oolite	flat	without unguent	0·49
Hard calcareous stone, or muschelkalk, upon muschelkalk . . .	ditto	ditto	0·70
Calcareous oolite stone upon muschelkalk .	ditto	ditto	0·75
Brick upon muschelkalk	ditto	ditto	0·67
Iron upon muschelkalk .	ditto	ditto	0·42
Oak upon muschelkalk .	ditto	ditto	0·64
Calcareous oolite stone upon calcareous oolite	ditto	with a coating of mortar, of three parts of fine sand, and one part of slack lime	0·74

Table *of* **Friction** *of* **Plane Surfaces** *in motion one upon the other.*

Surfaces in contact.	Disposition of the Fibres.	State of the Surfaces.	Co-efficient of Friction.
Experiments of M. Morin.			
Oak upon oak	parallel	without unguent	0·48
	ditto	rubbed with dry soap	0·16
	perpendicular	without unguent	0·34
	ditto	steeped in water	0·25
	wood endways on wood lengthways	without unguent	0·19
Elm upon oak	parallel	ditto	0·43
	perpendicular	ditto	0·45
	parallel	ditto	0·25
Ash, fir, beech, wild pear-tree, and service-tree, upon oak	ditto	ditto	0·36 to 0·40
Iron upon oak	ditto	ditto	0·62
		with water	0·26
		rubbed with dry soap	0·21
		without unguent	0·49
Cast-iron upon oak . .	ditto	with water	0·22
		rubbed with dry soap	0·19
Copper upon oak . . .	ditto	without unguent	0·62
Iron upon elm	ditto	ditto	0·25
Cast-iron upon elm . .	ditto	ditto	0·20
Black dressed leather upon oak	ditto	ditto	0·27
Tanned leather upon oak	flat, or lengthways and edgeways	ditto	0·30 to 0·35
		with water	0·29
Tanned leather upon cast-iron and brass .	ditto	without unguent	0·56
		steeped in water	0·36
		greased and steeped in water	0·23
		with oil	0·15

TABLE *of* FRICTION *of* PLANE SURFACES—(*continued.*)

Surfaces in contact.	Disposition of the Fibres.	State of the Surfaces.	Co-efficient of Friction.
EXPERIMENTS OF M. MORIN —*continued.*			
Hemp, in threads or in cord, upon oak . . .	parallel	without unguent	0·52
	perpendicular	with water	0·33
Oak and elm upon cast-iron	parallel	without unguent	0·38
Wild pear-tree, ditto .	ditto	ditto	0·44
Iron upon iron . . .	ditto	ditto	0·44
Iron upon cast-iron and brass	ditto	ditto	0·18
Cast-iron, ditto . . .	ditto	ditto	0·15
Brass upon brass .	ditto	ditto	0·20
Brass upon cast-iron .	ditto	ditto	0·22
Brass upon iron . .	ditto	ditto	0·16
Oak, elm, yoke elm, wild pear, cast-iron, wrought iron, steel, and moving one upon another, or on themselves . . .	ditto	greased in the usual way with tallow, hog's lard, oil, soft gom	0·07 to 0·08
		slightly greasy to the touch	0·15
Calcareous oolite stone upon calcareous oolite	ditto	without unguent	0 64
Calcareous stone, called muschelkalk, upon calcareous oolite . . .	ditto	ditto	0·67
Common brick upon calcareous oolite . . .	ditto	ditto	0·65
Oak upon calcareous oolite	wood endways	ditto	0·38
Wrought-iron ditto . .	parallel	ditto	0·69
Calcareous stone, called muschelkalk, upon muschelkalk	ditto	ditto	0·38
Calcareous oolite stone upon muschelkalk . .	ditto	ditto	0·65
Common brick, ditto .	ditto	ditto	0·60
Oak upon muschelkalk	wood endways	ditto	0·38
Iron upon muschelkalk	parallel	ditto	0·24
		saturated with water	0·30

TABLE *of* FRICTION *of* GUDGEONS *or* AXLE ENDS, *in* MOTION *upon their* BEARINGS. (*From the Experiments of Morin.*)

Surfaces in contact.	State of the Surfaces.	Co-efficient of Friction when the Grease is renewed.	
		In the usual Way.	Continuously.
Cast-iron axles in cast-iron bearings . .	coated with oil of olives, with hog's lard, tallow, and soft gom	0·07 to 0·08	0·054
	with the same, and water	0·08	0·28
	coated with asphaltum	0·054	0·19
	greasy	0·14	
	greasy and wetted	0·14	
Cast-iron axles, ditto . . .	coated with oil of olives, with hog's lard, tallow, and soft gom	0·07 to 0·08	0·054
	greasy	0·16	
	greasy and damped	0·16	
	scarcely greasy	0·19	
Cast-iron axles in lignum-vitæ bearings	without unguent	0·18	
	with oil or hog's lard	—	0·09
	greasy, with ditto	0·10	
	greasy, with a mixture of hog's lard and molybdæna	0·14	
Wrought-iron axles in cast-iron bearings	coated with oil of olives, tallow, hog's lard, or soft gom	0·07 to 0·08	0·054
Iron axles in brass bearings	coated with oil of olives, hog's lard, or tallow	0·07 to 0·08	0·054
	coated with hard gom	0·09	
	greasy and wetted	0·19	
	scarcely greasy	0·25	
Iron axles in lignum-vitæ bearings . .	coated with oil, or hog's lard	0·11	
	greasy	0·19	
Brass axles in brass bearings	coated with oil	0·10	
	with hog's lard	0·09	
Brass axles in cast-iron bearings	coated with oil or tallow	—	0·045 to 0·052
Lignum-vitæ axles, ditto .	coated with hog's lard	0·12	
	greasy	0·15	
Lignum-vitæ axles in lignum-vitæ bearings . .	coated with hog's lard	—	0·07

M.	0°	1°	2°	3°	4°	5°	M.
0	·0000000	·00015	·00061	·00137	·00244	·00381	0
1	0001	16	62	39	46	83	1
2	0002	16	63	·00140	48	86	2
3	0004	17	64	42	·00250	88	3
4	0007	17	65	43	52	·00391	4
5	0011	18	66	45	54	93	5
6	0015	18	67	46	56	96	6
7	0021	19	68	48	58	98	7
8	0027	·00020	69	·00150	·00260	·00401	8
9	0034	·00020	·00070	51	62	04	9
10	0042	21	71	53	64	06	10
1	0051	21	73	54	66	09	1
2	0061	22	74	56	69	·00412	2
3	0072	23	75	58	·00271	14	3
4	0083	23	76	59	73	17	4
5	0095	24	77	·00161	75	·00420	5
6	·0000108	24	78	62	77	22	6
7	0122	25	79	64	79	25	7
8	0137	26	·00081	66	·00281	28	8
9	0153	26	82	68	84	·00430	9
20	0170	27	83	69	86	33	20
1	0187	28	84	·00171	88	36	1
2	·0000205	28	85	73	·00290	38	2
3	0224	29	87	74	93	·00441	3
4	0244	·00030	88	76	95	44	4
5	0265	31	89	78	97	47	5
6	0287	31	·00090	·00179	99	49	6
7	·0000309	32	91	81	·00301	·00452	7
8	0332	33	93	83	04	55	8
9	0356	34	94	85	06	58	9
30	0381	34	95	87	08	·00460	30
1	·0000407	35	96	88	·00311	63	1
2	0434	36	98	·00190	13	66	2
3	0461	37	99	92	15	69	3
4	0489	37	·00100	94	17	·00472	4
5	·0000518	38	02	96	·00320	74	5
6	0548	39	03	97	22	77	6
7	0579	·00040	04	99	24	·00480	7
8	·0000611	41	06	·00201	27	83	8
9	0644	41	07	03	29	86	9
40	0677	42	08	05	·00332	89	40
1	·0000711	43	·00110	07	34	·00492	1
2	0746	44	11	08	36	94	2
3	0782	45	12	·00210	39	97	3
4	·0000819	46	14	12	·00341	·00500	4
5	0857	47	15	14	43	03	5
6	0896	48	17	16	46	06	6
7	·0000935	48	18	18	48	09	7
8	0975	49	19	·00220	·00351	·00512	8
9	·0001016	·00050	·00121	22	53	15	9
50	1058	51	22	24	56	18	50
1	·0001101	52	24	26	58	·00521	1
2	1145	53	25	28	·00361	24	2
3	1189	54	27	·00230	63	27	3
4	·0001234	55	28	32	65	·00530	4
5	1280	56	·00130	34	68	33	5
6	·0001327	57	31	36	·00370	36	6
7	1375	58	33	38	73	39	7
8	·0001423	59	34	·00240	75	·00542	8
9	1473	·00060	36	42	78	45	9
60	·0001523	61	37	44	·00381	48	60

M.	6°	7°	8°	9°	10°	11°	M.
0	·0055	·0075	·0097	·0123	·0152	·0184	0
1	55	75	98	24	52	84	1
2	55	75	98	24	53	85	2
3	56	76	99	24	53	85	3
4	56	76	99	25	54	86	4
5	56	76	99	25	54	87	5
6	57	77	·0100	26	55	87	6
7	57	77	·0100	26	55	88	7
8	57	77	01	27	56	88	8
9	58	78	01	27	57	89	9
10	58	78	01	28	57	89	10
1	58	78	02	28	58	·0190	1
2	58	79	02	29	58	·0190	2
3	59	79	03	29	59	91	3
4	59	·0080	03	·0130	59	92	4
5	59	·0080	03	·0130	·0160	92	5
6	·0060	·0080	04	31	·0160	93	6
7	·0060	81	04	31	61	93	7
8	·0060	81	05	31	61	94	8
9	61	81	05	32	62	94	9
20	61	82	06	32	62	95	20
1	61	82	06	33	63	96	1
2	62	83	06	33	63	96	2
3	62	83	07	34	64	97	3
4	62	83	07	34	64	97	4
5	63	84	08	35	65	98	5
6	63	84	08	35	65	98	6
7	63	84	09	36	66	99	7
8	64	85	09	36	66	·0200	8
9	64	85	09	37	67	·0200	9
30	64	86	·0110	37	67	01	30
1	65	86	·0110	38	68	01	1
2	65	86	11	38	69	02	2
3	65	87	11	39	69	02	3
4	66	87	12	39	·0170	03	4
5	66	87	12	·0140	·0170	04	5
6	66	88	12	·0140	71	04	6
7	67	88	13	41	71	05	7
8	67	89	13	41	72	05	8
9	67	89	14	41	72	06	9
40	68	89	14	42	73	07	40
1	68	·0090	15	42	73	07	1
2	68	·0090	15	43	74	08	2
3	69	91	16	43	74	08	3
4	69	91	16	44	75	09	4
5	69	91	16	44	75	·0210	5
6	·0070	92	17	45	76	·0210	6
7	·0070	92	17	45	77	11	7
8	·0070	93	18	46	77	11	8
9	71	93	18	46	78	12	9
50	71	93	19	47	78	13	50
1	71	94	19	47	79	13	1
2	72	94	·0120	48	79	14	2
3	72	95	·0120	48	·0180	14	3
4	72	95	·0120	49	·0180	15	4
5	73	95	21	49	81	16	5
6	73	96	21	·0150	82	16	6
7	73	96	22	·0150	82	17	7
8	74	97	22	51	83	17	8
9	74	97	23	51	83	18	9
60	75	97	23	52	84	19	60

M.	12°	13°	14°	15°	16°	17°	M.
0	·0219	·0256	·0297	·0341	·0387	·0437	0
1	19	57	98	41	88	38	1
2	·0220	58	98	42	89	39	2
3	·0220	58	99	43	·0390	·0440	3
4	21	59	·0300	44	91	·0440	4
5	22	·0260	01	45	91	41	5
6	22	·0260	01	45	92	42	6
7	23	61	02	46	93	43	7
8	23	62	03	47	94	44	8
9	24	62	03	48	95	45	9
10	25	63	04	48	95	45	10
1	25	64	05	49	96	46	1
2	26	64	06	·0350	97	47	2
3	26	65	06	51	98	48	3
4	27	66	07	51	99	49	4
5	28	66	08	52	·0400	·0450	5
6	28	67	08	53	·0400	51	6
7	29	68	09	54	01	52	7
8	·0230	68	·0310	54	02	52	8
9	·0230	69	11	55	03	53	9
20	31	·0270	11	56	04	54	20
1	31	·0270	12	57	04	55	1
2	32	71	13	58	05	56	2
3	33	72	13	58	06	57	3
4	33	72	14	59	07	58	4
5	34	73	15	·0360	08	58	5
6	35	74	16	61	09	59	6
7	35	74	16	61	09	·0460	7
8	36	75	17	62	·0410	61	8
9	36	76	18	63	11	62	9
30	37	76	19	64	12	63	30
1	38	77	19	64	13	64	1
2	38	78	·0320	65	13	65	2
3	39	78	21	66	14	65	3
4	·0240	79	21	67	15	66	4
5	·0240	·0280	22	68	16	67	5
6	41	·0280	23	68	17	68	6
7	41	81	24	69	18	69	7
8	42	82	24	·0370	18	·0470	8
9	43	82	25	71	19	71	9
40	43	83	26	72	·0420	72	40
1	44	84	27	72	21	73	1
2	45	85	27	73	22	73	2
3	45	85	28	74	23	74	3
4	46	86	29	75	23	75	4
5	47	87	·0330	75	24	76	5
6	47	87	·0330	76	25	77	6
7	48	88	31	77	26	78	7
8	49	89	32	78	27	79	8
9	49	89	33	79	28	·0480	9
50	·0250	·0290	33	79	28	·0480	50
1	·0250	91	34	·0380	29	81	1
2	51	91	35	81	·0430	82	2
3	52	92	35	82	31	83	3
4	52	93	36	83	32	84	4
5	53	94	37	83	33	85	5
6	54	94	38	84	34	86	6
7	54	95	38	85	34	87	7
8	55	96	39	86	35	88	8
9	56	96	·0340	87	36	89	9
60	56	97	41	87	37	89	60

M.	18°	19°	20°	21°	22°	23°	M.
0	·0489	·0545	·0603	·0664	·0728	·0795	0
1	·0490	46	04	65	29	96	1
2	91	47	05	66	·0730	97	2
3	92	48	06	67	31	98	3
4	93	49	07	68	33	·0800	4
5	94	·0550	08	69	34	01	5
6	95	51	09	·0670	35	02	6
7	96	51	·0610	72	36	03	7
8	97	52	11	73	37	04	8
9	98	53	12	74	38	05	9
10	98	54	13	75	39	06	10
1	99	55	14	76	·0740	08	1
2	·0500	56	15	77	41	09	2
3	01	57	16	78	42	·0810	3
4	02	58	17	79	43	11	4
5	03	59	18	·0680	45	12	5
6	04	·0560	19	81	46	13	6
7	05	61	·0620	82	47	14	7
8	06	62	21	83	48	16	8
9	07	63	22	84	49	17	9
20	08	64	23	85	·0750	18	20
1	08	65	24	86	51	19	1
2	09	66	25	87	52	·0820	2
3	·0510	67	26	88	53	21	3
4	11	68	27	89	55	22	4
5	12	69	28	·0691	56	24	5
6	13	·0570	29	92	57	25	6
7	14	71	·0630	93	58	26	7
8	15	72	31	94	59	27	8
9	16	73	32	95	·0761	28	9
30	17	74	33	96	61	29	30
1	18	75	34	97	62	·0831	1
2	19	76	35	98	63	32	2
3	·0520	77	36	99	65	33	3
4	·0520	77	37	·0700	66	34	4
5	21	78	38	01	67	35	5
6	22	79	39	02	68	36	6
7	23	·0580	·0640	03	69	38	7
8	24	81	41	04	·0770	39	8
9	25	82	42	05	71	·0840	9
40	26	83	44	07	72	41	40
1	27	84	45	08	73	42	1
2	28	85	46	09	75	43	2
3	29	86	47	·0710	76	45	3
4	·0530	87	48	11	77	46	4
5	31	88	49	12	78	47	5
6	32	89	·0650	13	79	48	6
7	33	·0590	51	14	·0780	49	7
8	34	91	52	15	81	·0850	8
9	34	92	53	16	82	52	9
50	35	93	54	17	84	53	50
1	36	94	55	18	85	54	1
2	37	95	56	19	86	55	2
3	38	96	57	·0721	87	56	3
4	39	97	58	22	88	57	4
5	·0540	98	59	23	89	59	5
6	41	99	·0660	24	·0790	·0860	6
7	42	·0600	61	25	92	61	7
8	43	01	62	26	93	62	8
9	44	02	63	27	94	63	9
60	45	03	64	28	95	65	60

M.	24°	25°	26°	27°	28°	29°	M.
0	·0865	·0937	·1012	·1090	·1171	·1254	0
1	66	38	13	91	72	55	1
2	67	39	15	93	73	57	2
3	68	·0941	16	94	75	58	3
4	69	42	17	95	76	59	4
5	·0870	43	18	97	77	·1261	5
6	72	44	·1020	98	79	62	6
7	73	46	21	99	·1180	64	7
8	74	47	22	·1101	81	65	8
9	75	48	24	02	83	67	9
10	76	49	25	03	84	68	10
1	78	·0950	26	05	86	69	1
2	79	52	27	06	87	·1271	2
3	·0880	53	29	07	88	72	3
4	81	54	·1030	08	·1190	74	4
5	82	55	31	·1110	91	75	5
6	84	57	33	11	92	76	6
7	85	58	34	12	94	78	7
8	86	59	35	14	95	79	8
9	87	·0960	36	15	97	·1281	9
20	88	62	38	16	98	82	20
1	·0890	63	39	18	99	84	1
2	91	64	·1040	19	·1201	85	2
3	92	65	42	·1121	02	86	3
4	93	67	43	22	04	88	4
5	94	68	44	23	05	89	5
6	96	69	45	25	06	·1291	6
7	97	·0970	47	26	08	92	7
8	98	72	48	27	09	94	8
9	99	73	49	29	·1210	95	9
30	·0900	74	·1051	·1130	12	96	30
1	02	75	52	31	13	98	1
2	03	77	53	33	15	99	2
3	04	78	55	34	16	·1301	3
4	05	79	56	35	17	02	4
5	06	·0980	57	37	19	04	5
6	08	82	58	38	·1220	05	6
7	09	83	·1060	39	22	06	7
8	·0910	84	61	·1141	23	08	8
9	11	85	62	42	24	09	9
40	12	87	64	43	26	·1311	40
1	14	88	65	45	27	12	1
2	15	89	66	46	29	14	2
3	16	·0990	68	47	·1230	15	3
4	17	92	69	49	31	17	4
5	19	93	·1070	·1150	33	18	5
6	·0920	94	72	51	34	19	6
7	21	96	73	53	36	·1321	7
8	22	97	74	54	37	22	8
9	23	98	75	56	38	24	9
50	25	99	77	57	·1240	25	50
1	26	·1001	78	58	41	27	1
2	27	02	79	·1160	43	28	2
3	28	03	·1081	61	44	·1330	3
4	·0930	04	82	62	45	31	4
5	31	06	83	64	47	32	5
6	32	07	85	65	48	34	6
7	33	08	86	66	·1250	35	7
8	34	·1010	87	68	51	37	8
9	36	11	89	69	52	38	9
60	37	12	·1090	·1171	54	·1340	60

M.	30°	31°	32°	33°	34°	35°	M.
0	·1340	·1428	·1520	·1613	·1710	·1808	0
1	41	·1430	21	15	11	·1810	1
2	43	31	23	16	13	12	2
3	44	33	24	18	15	13	3
4	46	34	26	·1620	16	15	4
5	47	36	27	21	18	17	5
6	48	37	29	23	19	19	6
7	·1350	39	·1530	24	·1721	·1820	7
8	51	·1440	32	26	23	22	8
9	53	42	33	28	24	24	9
10	54	43	35	29	26	25	10
1	56	45	37	·1631	28	27	1
2	57	46	38	32	29	29	2
3	59	48	·1540	34	·1731	·1830	3
4	·1360	49	41	36	32	32	4
5	62	·1451	43	37	34	34	5
6	63	52	44	39	36	35	6
7	64	54	46	·1640	37	37	7
8	66	55	47	42	39	39	8
9	68	57	49	44	·1741	·1840	9
20	69	58	·1550	45	42	42	20
1	·1370	·1460	52	47	44	44	1
2	72	61	54	48	46	45	2
3	73	63	55	·1650	47	47	3
4	75	64	57	52	49	49	4
5	76	66	58	53	·1751	·1850	5
6	78	68	·1560	55	52	52	6
7	79	69	61	56	54	54	7
8	·1381	·1471	63	58	55	55	8
9	82	72	65	·1660	57	57	9
30	84	74	66	61	59	59	30
1	85	75	68	63	·1760	·1861	1
2	87	77	69	64	62	62	2
3	88	78	·1571	66	64	64	3
4	·1390	·1480	72	68	65	66	4
5	91	81	74	69	67	67	5
6	93	83	75	·1671	69	69	6
7	94	84	77	72	·1770	·1871	7
8	96	86	79	74	72	72	8
9	97	87	·1580	76	74	74	9
40	99	89	82	77	75	76	40
1	·1400	·1490	83	79	77	77	1
2	01	92	85	·1680	79	79	2
3	03	93	86	82	·1780	·1881	3
4	04	95	88	84	82	83	4
5	06	96	·1590	85	84	84	5
6	07	98	91	87	85	86	6
7	09	·1500	93	89	87	88	7
8	·1410	01	94	·1690	89	89	8
9	12	03	96	92	·1790	·1891	9
50	13	04	97	93	92	93	50
1	15	06	99	95	93	94	1
2	16	07	·1601	97	95	96	2
3	18	09	02	98	97	98	3
4	19	·1510	04	·1700	98	·1900	4
5	·1421	12	05	02	·1800	01	5
6	22	13	07	03	02	03	6
7	24	15	09	05	03	05	7
8	25	16	·1610	06	05	06	8
9	27	18	12	08	07	08	9
60	28	·1520	13	·1710	08	·1910	60

M.	36°	37°	38°	39°	40°	41°	M.
0	·1910	·2014	·2120	·2229	·2340	·2453	0
1	12	15	22	·2230	41	55	1
2	13	17	23	32	43	57	2
3	15	19	25	34	45	59	3
4	17	·2021	27	36	47	·2461	4
5	18	22	29	38	49	62	5
6	·1920	24	·2131	·2240	·2351	64	6
7	22	26	32	41	53	66	7
8	24	28	34	43	55	68	8
9	25	29	36	45	56	·2470	9
10	27	·2031	38	47	58	72	10
1	29	33	·2140	49	·2360	74	1
2	·1930	35	41	·2251	62	76	2
3	32	36	43	52	64	78	3
4	34	38	45	54	66	·2480	4
5	36	·2040	47	56	68	82	5
6	37	42	49	58	·2370	84	6
7	39	44	·2150	·2260	71	85	7
8	·1941	45	52	62	73	87	8
9	42	47	54	63	75	89	9
20	44	49	56	65	77	·2491	20
1	46	·2051	58	67	79	93	1
2	48	52	59	69	·2381	95	2
3	49	54	·2161	·2271	83	97	3
4	·1951	56	63	73	85	99	4
5	53	58	65	75	87	·2501	5
6	55	59	67	76	88	03	6
7	56	·2061	68	78	·2390	05	7
8	58	63	·2170	·2280	92	07	8
9	·1960	65	72	82	94	09	9
30	61	66	74	84	96	·2510	30
1	63	68	76	86	98	12	1
2	65	·2070	78	87	·2400	14	2
3	67	72	79	89	02	16	3
4	68	74	·2181	·2291	04	18	4
5	·1970	75	83	93	05	·2520	5
6	72	77	85	95	07	22	6
7	74	79	87	97	09	24	7
8	75	·2081	88	99	·2411	26	8
9	77	82	·2190	·2300	13	28	9
40	79	84	92	02	15	·2530	40
1	·1981	86	94	04	17	32	1
2	82	88	96	06	19	34	2
3	84	·2090	98	08	·2421	36	3
4	86	91	99	·2310	22	37	4
5	87	93	·2201	12	24	39	5
6	89	95	03	13	26	·2541	6
7	·1991	97	05	15	28	43	7
8	93	98	07	17	·2430	45	8
9	94	·2100	08	19	32	47	9
50	96	02	·2210	·2321	34	49	50
1	98	04	12	23	36	·2551	1
2	·2000	06	14	25	38	53	2
3	01	07	16	26	·2440	56	3
4	03	09	18	28	41	57	4
5	05	·2111	19	·2330	43	59	5
6	07	13	·2221	32	45	·2561	6
7	08	15	23	34	47	63	7
	·2010	16	25	36	49	65	8
	12	18	27	38	·2451	67	9
	14	·2120	29	·2340	53	69	60

M.	42°	43°	44°	45°	46°	47°	M.
0	·2569	·2686	·2807	·2929	·3053	·3180	0
1	·2570	88	09	·2931	56	82	1
2	72	·2690	·2811	33	58	84	2
3	74	92	13	35	·3060	86	3
4	76	94	15	37	62	89	4
5	78	96	17	39	64	·3191	5
6	·2580	98	19	·2941	66	93	6
7	82	·2700	·2821	43	68	95	7
8	84	02	23	45	·3070	97	8
9	86	04	25	47	72	99	9
10	88	06	27	·2950	74	·3201	10
1	·2590	08	29	52	76	03	1
2	92	·2710	·2831	54	79	06	2
3	94	12	33	56	·3081	08	3
4	96	14	35	58	83	·3210	4
5	98	16	37	·2960	85	12	5
6	·2600	18	39	62	87	14	6
7	02	·2720	·2841	64	89	16	7
8	04	22	43	66	·3091	18	8
9	06	24	45	68	93	·3221	9
20	08	26	47	·2970	95	23	20
1	·2610	28	49	72	97	25	1
2	12	·2730	·2851	74	·3100	27	2
3	13	32	53	76	02	29	3
4	15	34	55	78	04	·3231	4
5	17	36	57	·2981	06	33	5
6	19	38	59	83	08	36	6
7	·2621	·2740	·2861	85	·3110	38	7
8	23	42	63	87	12	·3240	8
9	25	44	65	89	14	42	9
30	27	46	67	·2991	16	44	30
1	29	48	·2870	93	19	46	1
2	·2631	·2750	72	96	·3121	48	2
3	33	52	74	97	23	·3251	3
4	35	54	76	99	25	53	4
5	37	56	78	·3001	27	55	5
6	39	58	·2880	03	29	57	6
7	·2641	·2760	82	05	·3131	59	7
8	43	62	84	08	33	·3261	8
9	45	64	86	·3010	35	63	9
40	47	66	88	12	38	66	40
1	49	68	·2890	14	·3140	68	1
2	·2651	·2770	92	16	42	·3270	2
3	53	72	94	18	44	72	3
4	55	74	96	·3020	46	74	4
5	57	76	98	22	48	76	5
6	59	78	·2900	24	·3150	78	6
7	·2661	·2780	02	26	52	·3281	7
8	63	82	04	28	55	83	8
9	65	84	06	·3030	57	85	9
50	67	86	08	33	59	87	50
1	69	88	·2910	35	·3161	89	1
2	·2671	·2790	12	37	63	·3291	2
3	73	92	15	39	65	94	3
4	75	94	17	·3041	67	96	4
5	77	97	19	43	69	98	5
6	79	99	·2921	45	·3172	·3300	6
7	·2681	·2801	23	47	74	02	7
8	82	03	25	49	76	04	8
9	84	05	27	·3051	78	07	9
60	86	07	29	53	·3180	09	60

M.	48°	49°	50°	51°	52°	53°	M.
0	·3309	·3439	·3572	·3707	·3843	·3982	0
1	·3311	·3442	74	09	46	84	1
2	13	44	77	·3711	48	86	2
3	15	46	79	14	·3850	89	3
4	17	48	·3581	16	53	·3991	4
5	·3320	·3450	83	18	55	93	5
6	22	53	86	·3720	57	96	6
7	24	55	88	23	59	98	7
8	26	57	·3590	25	·3862	·4000	8
9	28	59	92	27	64	03	9
10	·3330	·3461	94	29	66	05	10
1	33	64	97	·3732	69	07	1
2	35	66	99	34	·3871	·4010	2
3	37	68	·3601	36	73	12	3
4	39	·3470	03	38	76	14	4
5	·3341	72	06	·3741	78	17	5
6	43	75	08	43	·3880	19	6
7	46	77	·3610	45	82	·4021	7
8	48	79	12	48	85	24	8
9	·3350	·3481	15	·3750	87	26	9
20	52	83	17	52	89	28	20
1	54	86	19	54	·3892	·4031	1
2	56	88	·3621	57	94	33	2
3	59	·3490	24	59	96	35	3
4	·3361	92	26	·3761	99	38	4
5	63	94	28	63	·3901	·4040	5
6	65	97	·3630	66	03	42	6
7	67	99	32	68	05	45	7
8	69	·3501	35	·3770	08	47	8
9	·3372	03	37	73	·3910	49	9
30	74	06	39	75	12	·4052	30
1	76	08	·3641	77	15	54	1
2	78	·3510	44	79	17	56	2
3	·3380	12	46	·3782	19	59	3
4	83	14	48	84	·3922	·4061	4
5	85	17	·3650	86	24	63	5
6	87	19	53	89	26	66	6
7	89	·3521	55	·3791	29	68	7
8	·3391	23	57	93	·3931	·4070	8
9	93	25	59	95	33	73	9
40	96	28	·3662	98	35	75	40
1	98	·3530	64	·3800	38	78	1
2	·3400	32	66	02	·3940	·4080	2
3	02	34	68	04	42	82	3
4	04	37	·3671	07	45	85	4
5	07	39	73	09	47	87	5
6	09	·3541	75	·3811	49	89	6
7	·3411	43	77	14	·3952	·4092	7
8	13	45	·3680	16	54	94	8
9	15	48	82	18	56	96	9
50	17	·3550	84	·3820	59	99	50
1	·3420	52	86	23	·3961	·4101	1
2	22	54	89	25	63	03	2
3	24	57	·3691	27	66	06	3
4	26	59	93	·3830	68	08	4
5	28	·3561	96	32	·3970	·4110	5
6	·3431	63	98	34	73	13	6
7	33	65	·3700	37	75	15	7
8	35	68	02	39	77	17	8
9	37	·3570	05	·3841	·3980	·4120	9
60	39	72	07	43	82	22	60

M.	54°	55°	56°	57°	58°	59°	M.
0	·4122	·4264	·4408	·4554	·4701	·4850	0
1	25	67	·4410	56	03	52	1
2	27	69	13	58	06	55	2
3	29	·4271	15	·4561	08	57	3
4	·4132	74	18	63	·4711	·4860	4
5	34	76	·4420	66	13	62	5
6	36	79	23	68	16	65	6
7	39	·4281	25	·4571	18	67	7
8	·4141	83	27	73	·4721	·4870	8
9	43	86	·4430	76	23	72	9
10	46	88	32	78	25	75	10
1	48	·4290	35	·4580	28	77	1
2	·4150	93	37	83	·4730	·4880	2
3	53	95	39	85	33	82	3
4	55	98	·4442	88	35	85	4
5	58	·4300	44	·4590	38	87	5
6	·4160	02	47	93	·4740	·4890	6
7	62	05	49	95	43	92	7
8	65	07	·4452	98	45	95	8
9	67	·4310	54	·4600	48	97	9
20	69	12	56	02	·4750	·4900	20
1	·4172	14	59	05	53	02	1
2	74	17	·4461	07	55	05	2
3	76	19	64	·4610	58	07	3
4	79	·4322	66	12	·4760	·4910	4
5	·4181	24	69	15	63	12	5
6	84	26	·4471	17	65	15	6
7	86	29	73	·4620	68	17	7
8	88	·4331	76	22	·4770	·4920	8
9	·4191	34	78	25	73	22	9
30	93	36	·4481	27	75	25	30
1	95	38	83	29	77	27	1
2	98	·4341	85	·4632	·4780	·4930	2
3	·4200	43	88	34	82	32	3
4	02	46	·4490	37	85	35	4
5	05	48	93	39	87	37	5
6	07	·4350	95	·4642	·4790	·4940	6
7	·4210	53	98	44	92	42	7
8	12	55	·4500	47	95	45	8
9	14	58	02	49	97	47	9
40	17	·4360	05	·4652	·4800	·4950	40
1	19	62	07	54	02	52	1
2	·4221	65	·4510	56	05	55	2
3	24	67	12	59	07	57	3
4	26	·4370	15	·5661	·4810	·4960	4
5	29	72	17	64	12	62	5
6	·4231	74	·4520	66	15	65	6
7	33	77	22	69	17	67	7
8	36	79	24	·4671	·4820	·4970	8
9	38	·4382	27	74	22	72	9
50	·4240	84	29	76	25	75	50
1	43	86	·4532	79	27	77	1
2	45	89	34	·4681	·4830	·4980	2
3	48	·4391	37	84	32	82	3
4	·4250	94	39	86	35	85	4
5	52	96	·4541	88	37	87	5
6	55	98	44	·4691	·4840	·4990	6
7	57	·4401	46	93	42	92	7
8	59	03	49	96	45	95	8
9	·4262	06	·4551	98	47	97	9
60	64	08	54	·4701	·4850	·5000	60

M.	60°	61°	62°	63°	64°	65°	M.
0	·5000	·5152	·5305	·5460	·5616	·5774	0
1	03	54	08	63	19	76	1
2	05	57	·5310	65	·5622	79	2
3	08	·5160	13	68	24	·5782	3
4	·5010	62	16	·5470	27	84	4
5	13	65	18	73	29	87	5
6	15	67	·5321	76	·5632	·5790	7
7	18	·5170	23	78	35	92	6
8	·5020	72	26	·5481	37	95	8
9	23	75	28	83	·5640	98	9
10	25	77	·5331	86	42	·5800	10
1	28	·5180	34	89	45	03	1
2	·5030	82	36	·5491	48	05	2
3	33	85	39	94	·5650	08	3
4	35	88	·5341	96	53	·5811	4
5	38	·5190	44	99	56	13	5
6	·5040	93	46	·5502	58	16	6
7	43	95	49	04	·5661	19	7
8	45	98	·5352	07	63	·5821	8
9	48	·5200	54	09	66	24	9
20	·5050	03	57	·5512	69	27	20
1	53	05	59	15	·5671	29	1
2	56	08	·5362	17	74	·5832	2
3	58	·5211	64	·5520	77	35	3
4	·5061	13	67	22	79	37	4
5	63	16	·5370	25	·5682	·5840	5
6	66	18	72	28	84	42	6
7	68	·5221	75	·5530	87	45	7
8	·5071	23	77	33	·5690	48	8
9	73	26	·5380	35	92	·5850	9
30	76	28	83	38	95	53	30
1	78	·5231	85	·5541	98	56	1
2	·5081	34	88	43	·5700	58	2
3	83	36	·5390	46	03	·5861	3
4	86	39	93	48	05	64	4
5	88	·5241	95	·5551	08	66	5
6	·5091	44	98	54	·5711	69	6
7	93	46	·5401	56	13	·5872	7
8	96	49	03	59	16	74	8
9	99	·5251	06	·5561	19	77	9
40	·5101	54	08	64	·5721	·5880	40
1	04	57	·5411	67	24	82	1
2	06	59	14	69	26	85	2
3	09	·5262	16	·5572	29	88	3
4	·5111	64	19	75	·5732	·5890	4
5	14	67	·5421	77	34	93	5
6	16	69	24	·5580	37	95	6
7	19	·5272	26	82	·5740	98	7
8	·5121	74	29	85	42	·5901	8
9	24	77	·5432	88	45	03	9
50	26	·5280	34	·5590	47	06	50
1	29	82	37	93	·5750	09	1
2	·5132	85	39	95	53	·5911	2
3	34	87	·5442	98	55	14	3
4	37	·5290	45	·5601	58	17	4
5	39	92	47	03	·5761	19	5
6	·5142	95	·5450	06	63	·5922	6
7	44	98	52	08	66	25	7
8	47	·5300	55	·5611	69	27	8
9	49	03	58	14	·5771	·5930	9
60	·5152	05	·5460	16	74	33	60

M.	66°	67°	68°	69°	70°	71°	M.
0	·5933	·6093	·6254	·6416	·6580	·6744	0
1	35	95	57	19	83	47	1
2	38	98	59	·6422	85	·6750	2
3	·5941	·6101	·6262	24	88	53	3
4	43	03	65	27	·6591	55	4
5	46	06	67	·6430	93	58	5
6	49	09	·6270	33	96	·6761	6
7	·5951	·6111	73	35	99	64	7
8	54	14	76	38	·6602	66	8
9	57	17	78	·6441	04	69	9
10	59	19	·6281	43	07	·6772	10
1	·5962	·6122	84	46	·6610	75	1
2	65	25	86	49	13	77	2
3	67	28	89	·6452	15	·6780	3
4	·5970	·6130	·6292	54	18	83	4
5	73	33	94	57	·6621	86	5
6	75	36	97	·6460	24	88	6
7	78	38	·6300	63	26	·6791	7
8	·5981	·6141	03	65	29	94	8
9	83	44	05	68	·6632	97	9
20	86	46	08	·6471	35	99	20
1	89	49	·6311	73	37	·6802	1
2	·5991	·6152	13	76	·6640	05	2
3	94	54	16	79	43	08	3
4	97	57	19	·6482	45	·6810	4
5	99	·6160	·6321	84	48	13	5
6	·6002	62	24	87	·6651	16	6
7	05	65	27	·6490	54	19	7
8	07	68	·6330	92	56	·6821	8
9	·6010	·6170	32	95	59	24	9
30	13	73	35	98	·6662	27	30
1	15	76	38	·6501	65	·6830	1
2	18	79	·6340	03	67	32	2
3	·6021	·6181	43	06	·6670	35	3
4	23	84	46	09	73	38	4
5	26	87	49	·6512	76	·6841	5
6	29	89	·6351	14	78	44	6
7	·6031	·6192	54	17	·6681	46	7
8	34	95	57	·6520	84	49	8
9	37	97	59	22	87	·6852	9
40	39	·6200	·6362	25	89	55	40
1	·6042	03	65	28	·6692	57	1
2	45	05	67	·6531	95	·6860	2
3	47	08	·6370	33	98	63	3
4	·6050	·6211	73	36	·6700	66	4
5	53	14	76	39	03	68	5
6	55	16	78	·6542	06	·6871	6
7	58	19	·6381	44	09	74	7
8	·6061	·6222	84	47	·6711	77	8
9	63	24	86	·6550	14	79	9
50	66	27	89	52	17	·6882	50
1	69	·6230	·6392	55	·6720	85	1
2	·6071	32	95	58	22	88	2
3	74	35	97	·6561	25	·6890	3
4	77	38	·6400	63	28	93	4
5	79	·6240	03	66	·6731	96	5
6	·6082	43	05	69	33	99	6
7	85	46	08	·6572	36	·6902	7
8	87	49	·6411	74	39	04	8
9	·6090	·6251	14	77	·6742	07	9
60	93	54	16	·6580	44	·6910	60

M.	72°	73°	74°	75°	76°	77°	M.
0	·6910	·7076	·7244	·7412	·7581	·7750	0
1	13	79	46	15	84	53	1
2	15	·7082	49	17	86	56	2
3	18	85	·7252	·7420	89	59	3
4	·6921	87	55	23	·7592	·7762	4
5	24	·7090	58	26	95	65	5
6	26	93	·7260	29	98	67	6
7	29	96	63	·7431	·7601	·7770	7
8	·6932	99	66	34	03	73	8
9	35	·7101	69	37	06	76	9
10	38	04	·7272	·7440	09	79	10
1	·6940	07	74	43	·7612	·7782	1
2	43	·7110	77	46	15	85	2
3	46	12	·7280	48	17	87	3
4	49	15	83	·7451	·7620	·7790	4
5	·6951	18	86	54	23	93	5
6	54	·7121	88	57	26	96	6
7	57	24	·7291	·7460	29	99	7
8	·6960	26	94	62	·7632	·7802	8
9	62	29	97	65	34	04	9
20	65	·7132	·7300	68	37	07	20
1	68	35	02	·7471	·7640	·7810	1
2	·6971	38	05	74	43	13	2
3	74	·7140	08	76	46	16	3
4	76	43	·7311	79	49	19	4
5	79	46	14	·7482	·7651	·7821	5
6	·6982	49	16	85	54	24	6
7	85	·7151	19	88	57	27	7
8	87	54	·7322	·7491	·7660	·7830	8
9	·6990	57	25	93	63	33	9
30	93	·7160	28	96	66	36	30
1	96	63	·7330	99	68	38	1
2	98	65	33	·7502	·7671	·7841	2
3	·7001	68	36	05	74	44	3
4	04	·7171	39	07	77	47	4
5	07	74	·7342	·7510	·7680	·7850	5
6	·7010	77	44	13	83	53	6
7	12	79	47	16	85	55	7
8	15	·7182	·7350	19	88	58	8
9	18	85	53	·7522	·7691	·7861	9
40	·7021	88	56	24	94	64	40
1	23	·7191	58	27	97	67	1
2	26	93	·7361	·7530	·7700	·7870	2
3	29	96	64	33	02	73	3
4	·7032	99	67	36	05	75	4
5	35	·7202	·7370	38	08	78	5
6	37	05	72	·7541	·7711	·7881	6
7	·7040	07	75	44	14	84	7
8	43	·7210	78	47	16	87	8
9	46	13	·7381	·7550	19	·7890	9
50	48	16	84	53	·7722	92	50
1	·7051	18	87	55	25	95	1
2	54	·7221	89	58	28	98	2
3	57	24	·7392	·7561	·7731	·7901	3
4	·7060	27	95	64	33	04	4
5	62	·7230	98	67	36	07	5
6	65	32	·7401	69	39	·7910	6
7	68	35	03	·7572	·7742	12	7
8	·7071	38	06	75	45	15	8
9	74	·7241	09	78	48	18	9
60	76	44	·7412	·7581	·7750	·7921	60

M.	78°	79°	80°	81°	82°	83°	M.
0	·7921	·8092	·8264	·8436	·8608	·8781	0
1	24	95	66	39	·8611	84	1
2	27	98	69	·8441	14	87	2
3	29	·8100	·8272	44	17	·8790	3
4	·7932	03	75	47	·8620	93	4
5	35	06	78	·8450	23	96	5
6	38	09	·8281	53	26	99	6
7	·7941	·8112	84	56	28	·8802	7
8	44	15	86	59	·8631	04	8
9	46	18	89	·8462	34	07	9
10	49	·8120	·8292	64	37	·8810	10
1	·7952	23	95	67	·8640	13	1
2	55	26	98	·8470	43	16	2
3	58	29	·8301	73	46	19	3
4	·7961	·8132	04	76	49	·8822	4
5	64	35	07	79	·8651	25	5
6	66	38	09	·8482	54	28	6
7	69	·8140	·8312	85	57	·8830	7
8	·7972	43	15	87	·8660	33	8
9	75	46	18	·8490	63	36	9
20	78	49	·8321	93	66	39	20
1	·7981	·8152	24	96	69	·8842	1
2	84	55	27	99	·8672	45	2
3	86	58	29	·8502	75	48	3
4	89	·8160	·8332	05	77	·8851	4
5	·7992	63	35	08	·8680	54	5
6	95	66	38	·8510	83	56	6
7	98	69	·8341	13	86	59	7
8	·8001	·8172	44	16	89	·8862	8
9	03	75	47	19	·8692	65	9
30	06	78	·8350	·8522	95	68	30
1	09	·8181	52	25	98	·8871	1
2	·8012	83	55	28	·8701	74	2
3	15	86	58	·8531	03	77	3
4	18	89	·8361	33	06	·8880	4
5	·8021	·8192	64	36	09	82	5
6	23	95	67	39	·8712	85	6
7	26	98	·8370	·8542	15	88	7
8	29	·8201	72	45	18	·8891	8
9	·8032	03	75	48	·8721	94	9
40	35	06	78	·8551	24	97	40
1	38	09	·8381	54	26	·8900	1
2	·8041	·8212	84	56	29	03	2
3	43	15	87	59	·8732	06	3
4	46	18	·8390	·8562	35	08	4
5	49	·8221	93	65	38	·8911	5
6	·8052	23	95	68	·8741	14	6
7	55	26	98	·8571	44	17	7
8	58	29	·8401	74	47	·8920	8
9	·8061	·8232	04	77	·8750	23	9
50	63	35	07	79	52	26	50
1	66	38	·8410	·8582	55	29	1
2	69	·8241	13	85	58	·8932	2
3	·8072	43	16	88	·8761	34	3
4	75	46	18	·8591	64	37	4
5	78	49	·8421	94	67	·8940	5
6	·8080	·8252	24	97	·8770	43	6
7	83	55	27	·8600	73	46	7
8	86	58	·8430	03	76	49	8
9	89	·8261	33	05	78	·8952	9
60	·8092	64	36	08	·8781	55	60

M.	84°	85°	86°	87°	88°	89°	M.
0	·8955	·9128	·9302	·9477	·9651	·9825	0
1	58	·9131	05	·9480	54	28	1
2	·8961	34	08	82	57	·9831	2
3	63	37	·9311	85	·9660	34	3
4	66	·9140	14	88	63	37	4
5	69	43	17	·9491	66	·9840	5
6	·8972	46	·9320	94	68	43	6
7	75	49	23	97	·9671	46	7
8	78	·9152	26	·9500	74	49	8
9	·8981	55	29	03	77	·9852	9
10	84	57	·9331	06	·9680	55	10
1	87	·9160	34	09	83	57	1
2	89	63	37	·9512	86	·9860	2
3	·8992	66	·9340	14	89	63	3
4	95	69	43	17	·9692	66	4
5	98	·9172	46	·9520	95	69	5
6	·9001	75	49	23	98	·9872	6
7	04	78	·9352	26	·9700	75	7
8	07	·9181	55	29	03	78	8
9	·9010	84	58	·9532	06	·9881	9
20	13	86	·9360	35	09	84	20
1	15	89	63	38	·9712	87	1
2	18	·9192	66	·9541	15	89	2
3	·9021	95	69	43	18	·9892	3
4	24	98	·9372	46	·9721	95	4
5	27	·9201	75	49	24	98	5
6	·9030	04	78	·9552	27	·9901	6
7	33	07	·9381	55	·9730	04	7
8	36	·9210	84	58	32	07	8
9	39	13	87	·9561	35	·9910	9
30	·9042	15	·9390	64	38	13	30
1	44	18	92	67	·9741	16	1
2	47	·9221	95	·9570	44	19	2
3	·9050	24	98	73	47	·9921	3
4	53	27	·9401	75	·9750	24	4
5	56	·9230	04	78	53	27	5
6	59	33	07	·9581	56	·9931	6
7	·9062	36	·9410	84	59	33	7
8	65	39	13	87	·9761	36	8
9	68	·9242	16	·9590	64	39	9
40	·9071	44	19	93	67	·9942	40
1	73	47	·9421	96	·9770	45	1
2	76	·9250	24	99	73	48	2
3	79	53	27	·9602	76	·9951	3
4	·9082	56	·9430	04	79	53	4
5	85	59	33	07	·9782	56	5
6	88	·9262	36	·9610	85	59	6
7	·9091	65	39	13	88	·9962	7
8	94	68	·9442	16	·9791	65	8
9	97	·9271	45	19	93	68	9
50	99	73	48	·9622	96	·9971	50
1	·9102	76	·9450	25	99	74	1
2	05	79	53	28	·9802	77	2
3	08	·9282	56	·9631	05	·9980	3
4	·9111	85	59	34	08	83	4
5	14	88	·9462	36	·9811	85	5
6	17	·9291	65	39	14	88	6
7	·9120	94	68	·9642	17	·9991	7
8	23	97	·9471	45	·9820	94	8
9	26	·9300	74	48	23	97	9
60	28	02	77	·9651	25	1·0000	60

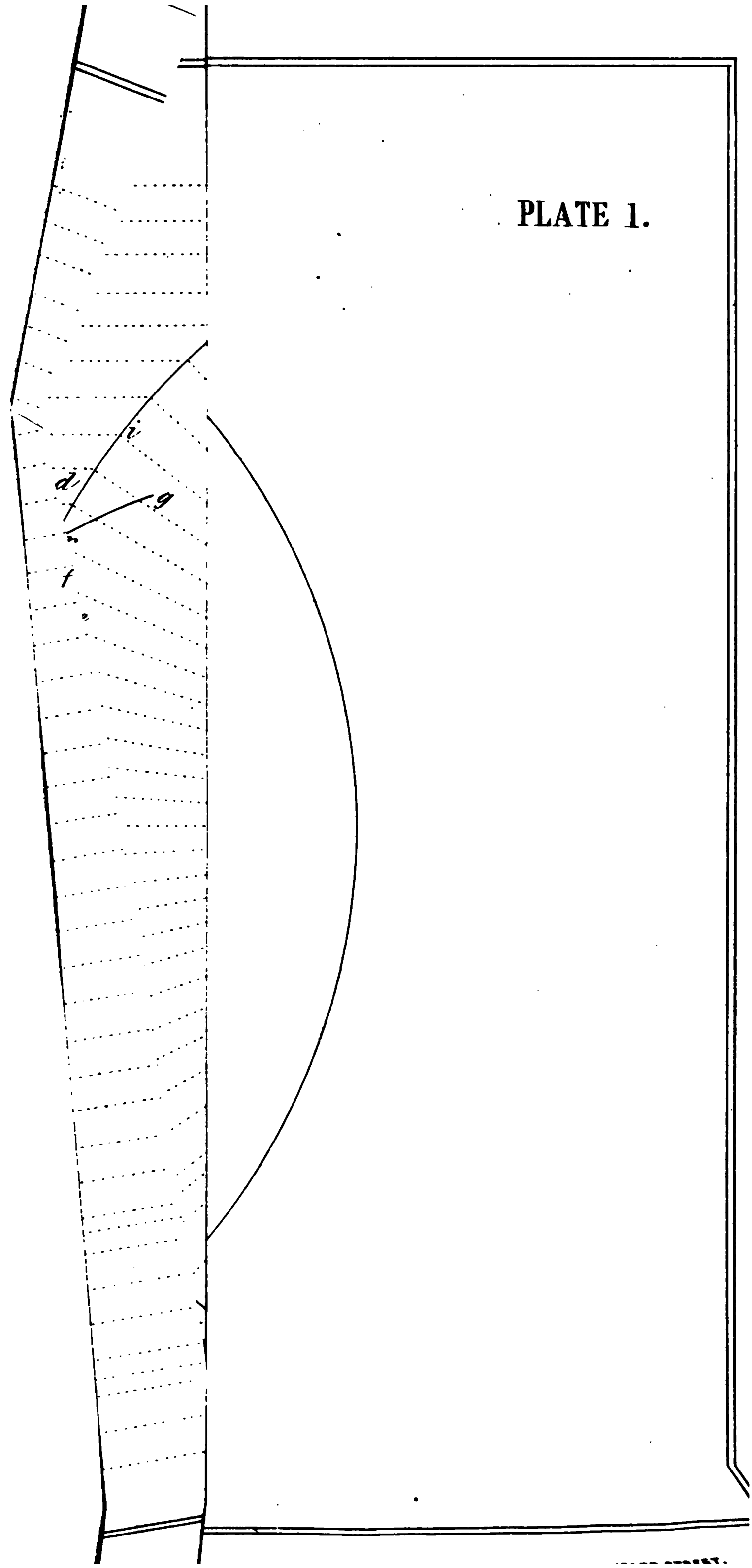
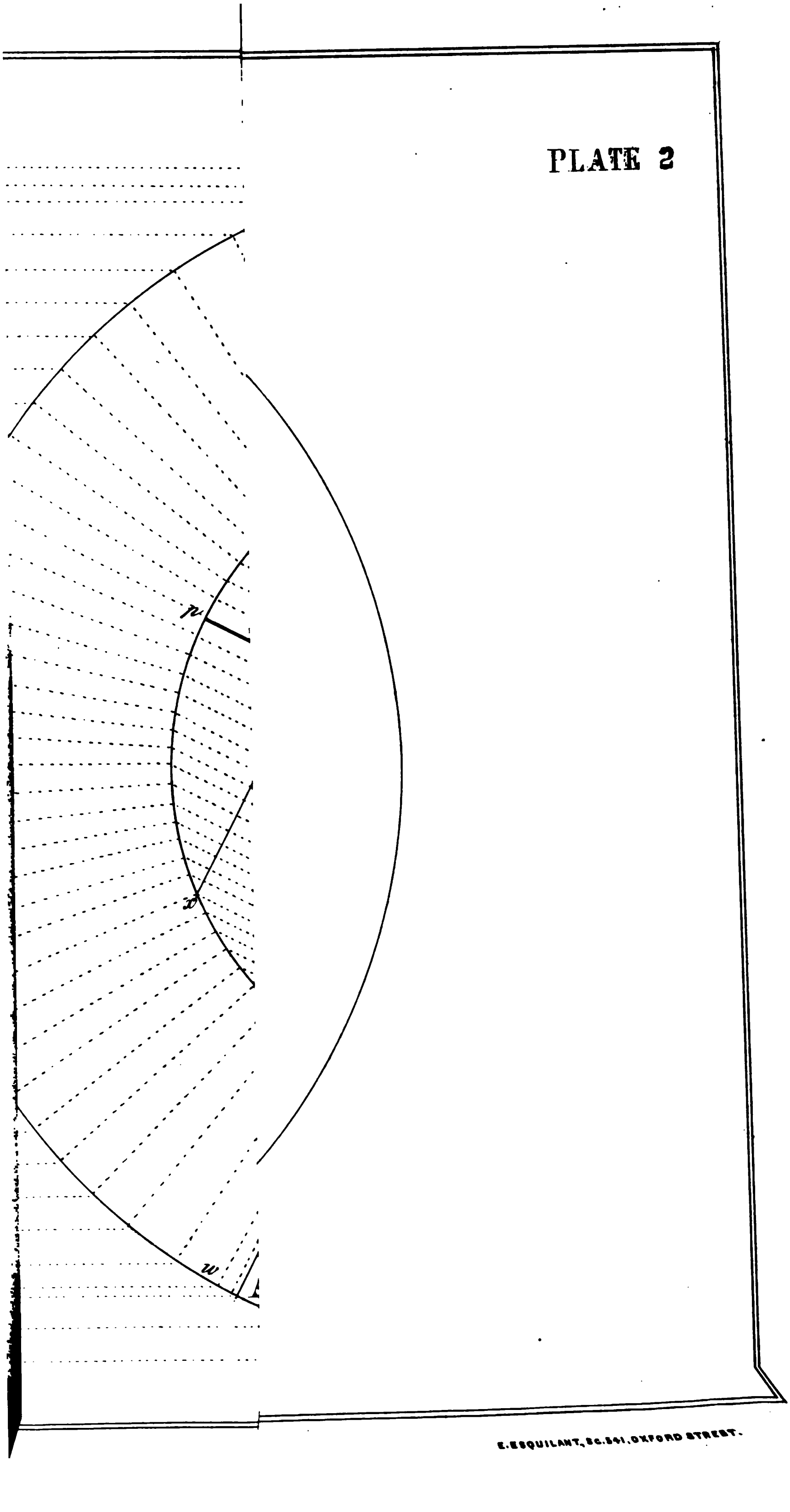


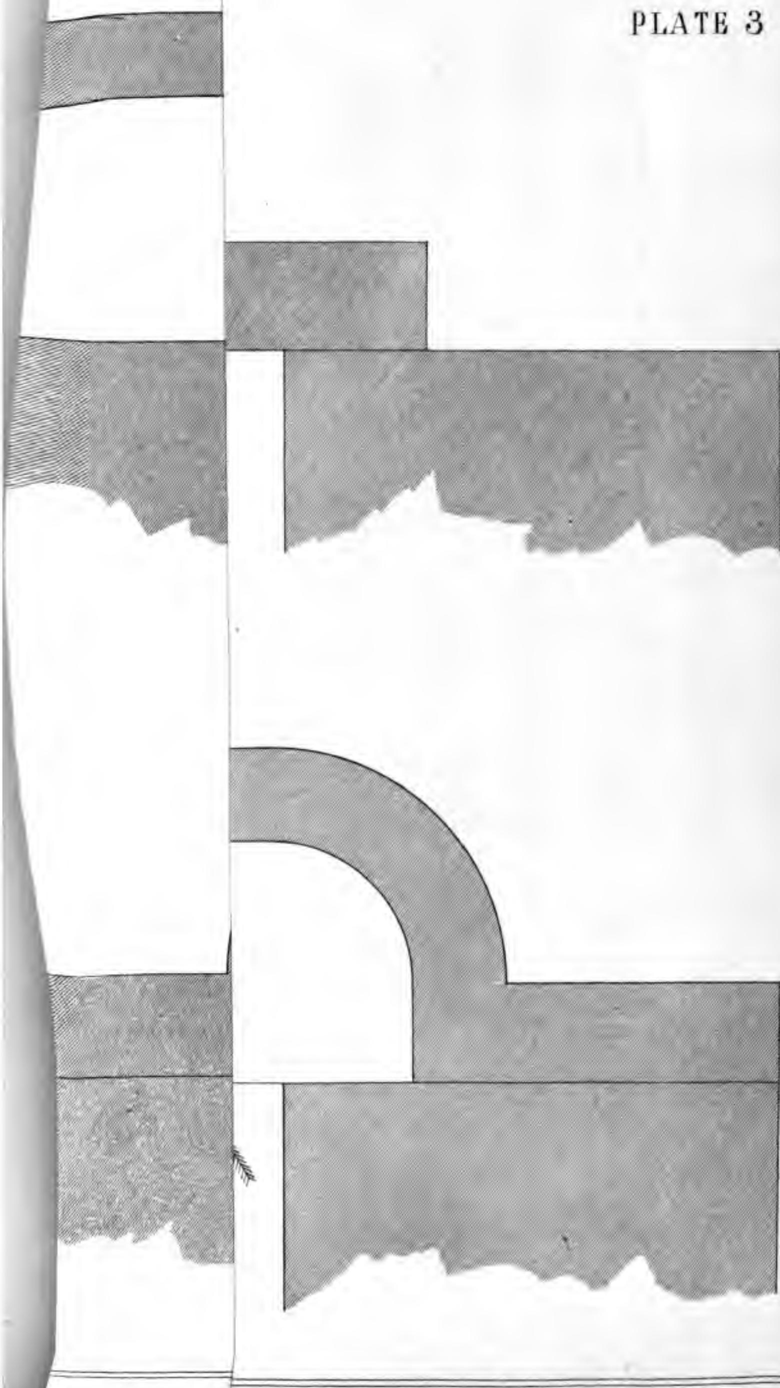
PLATE 1.

PLATE 2



E. ESQUILANT, Sc. 341, OXFORD STREET.

PLATE 3



E. ESQUILANT, Sc 541, OXFORD STREET